Technical Appendix: The Timing of Redistribution

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Abstract

This appendix contains material that was not included in the main paper due to space restrictions.

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1 Solving the Model

1.1 Households

An individual agent's maximization problem depends on whether she receives transfers or not. This will of course depend on the agent's initial ability level which in turn determines her income. We will have to calculate threshold ability levels θ_E^* and θ_L^* that determine whether an agent is poor enough to receive means tested early or late transfers, respectively. Whenever the initial ability/income level is below the threshold $\theta < \theta_E^*$, then the agent's labor income is low enough in order for early transfers to be positive, $T^E(\theta) > 0$. Whenever the agent's innate ability is equal or above this threshold, $\theta \geq \theta_E^*$ then early transfers will be zero, $T^E(\theta) = 0$. Similar conditions on ability hold for transfers in the second period. When second period wealth is low enough baed on the agent's ability $\theta < \theta_L^*$, then late transfers are positive, $T^L(\theta) > 0$. Whenever the initial ability is equal or above this threshold, $\theta \geq \theta_L^*$, then late transfers will be zero, $T^L(\theta) = 0$. This will generate four agent types, type one receives transfers when young and old, type two receives transfers when young only, type three receives transfers when old only, and type four does not receive any transfers.

Before substituting the budget constraints we first find the optimal relation between consumption and leisure. The ratio of marginal utilities has to equal the price ratio when solutions for leisure are interior, i.e. for $l \in (0, 1)$ it holds that

$$\frac{(1-\eta)c_t}{\eta l_t} = (1-b_E)\left(1-\tau^L\right)w_t\theta.$$

We can now express leisure in terms of consumption as

$$l_t = \min\left(\Theta_t(\theta)\, c_t, 1\right), \tag{1}$$

where $\Theta_t(\theta) = \frac{((1-\eta)/\eta)}{(1-b_E)(1-\tau^L)w_t\theta}$. Leisure is restricted to be within zero and one. We will next solve for the interior solution and then for the corner solution where leisure equals one.

1.1.1 Interior Solution for Leisure

We use expression (1) to substitute leisure out of the household budget constraint of the young individual and get

$$c_t + s_t = (1-b_E)\left(1-\tau^L\right)w_t\theta\left(1 - \frac{((1-\eta)/\eta)}{(1-b_E)\left(1-\tau^L\right)w_t\theta}c_t\right) + a_E.$$

After some simplifications this becomes

$$p_c c_t + s_t = (1-b_E)\left(1-\tau^L\right)w_t\theta + a_E,$$

where $p_c = \left(1 + \frac{1-\eta}{\eta}\right)$. Finally, we substitute leisure out of the preferences using (1) and get the preferences for the first period in consumption goods only as

$$u(c_t, l_t) = \frac{\left(c_t^{\eta}\left(\Theta_t(\theta)\, c_t\right)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} = \chi_t(\theta)\frac{c_t^{(\eta+1-\eta)(1-\sigma)}}{1-\sigma},$$

where $\chi_t(\theta) = \Theta_t(\theta)^{(1-\eta)(1-\sigma)}$. Now the maximization problem can be written as

$$\max_{\{c_t, c_{t+1}, s_t\}} \left\{ \chi_t(\theta) \frac{c_t^{(1-\sigma)}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right\} s.t.$$

$$\theta < \theta_E^* \wedge \theta < \theta_L^* : \begin{cases} p_c c_t + s_t = (1 - b_E)\left(1 - \tau^L\right)\theta w_t + a_E, \\ c_{t+1} = (1 - b_L) R_{t+1} s_t + a_L, \end{cases}$$

$$\theta < \theta_E^* \wedge \theta \geq \theta_L^* : \begin{cases} p_c c_t + s_t = (1 - b_E)\left(1 - \tau^L\right)\theta w_t + a_E, \\ c_{t+1} = R_{t+1} s_t, \end{cases}$$

$$\theta \geq \theta_E^* \wedge \theta < \theta_L^* : \begin{cases} p_c c_t + s_t = \left(1 - \tau^L\right)\theta w_t, \\ c_{t+1} = (1 - b_L) R_{t+1} s_t + a_L, \end{cases}$$

$$\theta \geq \theta_E^* \wedge \theta \geq \theta_L^* : \begin{cases} p_c c_t + s_t = \left(1 - \tau^L\right)\theta w_t, \\ c_{t+1} = R_{t+1} s_t, \end{cases}$$

where $R_t = \left(1 + \left(1 - \tau^K\right) r_t\right)$ and $\left(1 - \tau^K\right) r_t$ is the after tax interest rate. Households take the function relating transfers to savings into consideration. This results in the following decision rule for private savings

$$\theta < \theta_E^* \wedge \theta < \theta_L^* : s_t(\theta) = \frac{\left[(1 - b_E)\left(1 - \tau^L\right)\theta w_t + a_E\right]\left[\frac{p_c \beta (1 - b_L) R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}} - p_c a_L}{p_c (1 - b_L) R_{t+1} + \left[\frac{p_c \beta (1 - b_L) R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}, \quad (2)$$

$$\theta < \theta_E^* \wedge \theta \geq \theta_L^* : s_t(\theta) = \frac{\left[(1 - b_E)\left(1 - \tau^L\right)\theta w_t + a_E\right]\left[\frac{p_c \beta R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}{p_c R_{t+1} + \left[\frac{p_c \beta R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}, \quad (3)$$

$$\theta \geq \theta_E^* \wedge \theta < \theta_L^* : s_t(\theta) = \frac{\left(1 - \tau^L\right)\theta w_t \left[\frac{p_c \beta (1 - b_L) R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}} - p_c a_L}{p_c (1 - b_L) R_{t+1} + \left[\frac{p_c \beta (1 - b_L) R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}, \quad (4)$$

$$\theta \geq \theta_E^* \wedge \theta \geq \theta_L^* : s_t(\theta) = \frac{\left(1 - \tau^L\right)\theta w_t \left[\frac{p_c \beta R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}{p_c R_{t+1} + \left[\frac{p_c \beta R_{t+1}}{\chi_t(\theta)}\right]^{\frac{1}{\sigma}}}. \quad (5)$$

The threshold level of θ_E^* and θ_L^* that determine whether the household will receive targeted transfers are determined by the respective payout functions $T_t^E(\theta)$ and $T_t^L(\theta)$ and can be expressed as implicit function system

$$a_E - b_E\left(1 - \tau^L\right)\left(1 - l_t\left(\theta_E^*, \theta_L^*\right)\right)\theta_E^* w_t = 0, \text{ and} \quad (6)$$

$$a_L - b_L R_{t+1} s_t\left(\theta_E^*, \theta_L^*\right) = 0. \quad (7)$$

1.1.2 Corner Solution for Leisure

We next check the corner case for leisure $l = 1$. Preferences of the young agent will then reduce to

$$u(c_t, l_t = 1) = \frac{c_t^{\eta(1-\sigma)}}{1-\sigma},$$

and the maximization problem reduces to

$$\begin{aligned} & a_E < T_t^{e*} : \max_{\{c_t,\ c_{t+1},\ s_t\}} \left\{ \frac{c_t^{\eta(1-\sigma)}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right\} \quad (8) \\ & s.t. \\ c_t + s_t &= a_E, \\ c_{t+1} &= (1-b_L) R_{t+1} s_t + a_L, \end{aligned}$$

for the case where early transfers are low enough so that the agent is eligible to receive late transfers as well. If early transfers are beyond a certain level, then the agent is too rich and is not eligible for late transfers anymore and her problem reduces to

$$\begin{aligned} & a_E \geq T_t^{e*} : \max_{\{c_t,\ c_{t+1},\ s_t\}} \left\{ \frac{c_t^{\eta(1-\sigma)}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right\} \\ & s.t. \\ c_t + s_t &= a_E, \\ c_{t+1} &= R_{t+1} s_t, \end{aligned}$$

where T_t^{e*} is a similar threshold as the one above which determines whether the income (from transfer income) of the young is small enough, so that they have low enough wealth to be eligible for transfers when old. Note that the agent income does not depend on her ability type θ anymore as the agent decides to not work at all. Her only source of income are the early transfers from the government. If this early transfer is small enough, the agent will be eligible to receive transfers when old as well. Substituting the budget constraints in the objective functions we get again first order conditions of the form

$$\begin{aligned} a_E &< T^{E*} : \eta (a_E - s_t)^{\eta(1-\sigma)-1} = \beta (1-b_L) R_{t+1} ((1-b_L) R_{t+1} s_t + a_L)^{-\sigma}, \\ a_E &\geq T^{E*} : \eta (a_E - s_t)^{\eta(1-\sigma)-1} = \beta R_{t+1} (R_{t+1} s_t)^{-\sigma}. \end{aligned}$$

In this case we cannot get an explicit expression for savings. We therefore use implicit functions of the following form

$$a_E < T^{E*} : F(s_t) \equiv \eta (a_E - s_t)^{\eta(1-\sigma)-1} - \frac{\beta (1-b_L) R_{t+1}}{((1-b_L) R_{t+1} s_t + a)^{\sigma}} = 0, \quad (9)$$

$$a_E \geq T^{E*} : F(s_t) \equiv \eta (a_E - s_t)^{\eta(1-\sigma)-1} - \beta R_{t+1} (R_{t+1} s_t)^{-\sigma} = 0, \quad (10)$$

to determine the optimal amount of savings and solve for savings numerically. We can then solve for the threshold transfer level T^{E*} that determines whether the old agent receives a targeted transfer. The criterion is again derived from the payout formula for late transfers, expression $T_t^L(\theta)$ and can be written as implicit function

$$G\left(T_t^{E*}\right) \equiv a - bR_{t+1}s_t^*\left(T_t^e\right) = 0.$$

1.2 Government

Using the first order conditions from the firm's problem we can simplify the expression for total tax revenues $Tax_t = \left[\tau^L \int_\Theta w_t h_t(\theta)\, dF(\theta) + \tau^K \int_\Theta r_t s_{t-1}(\theta)\, dF(\theta)\right]$ to

$$Tax = \left[\tau^L(1-\alpha) + \tau^K \alpha\right] AK_t^\alpha H_t^{1-\alpha} - \tau^K \delta K,$$

and the two government budget constraints reduce to

$$\int_{\underline{\theta}}^{\theta_E^*} \left[a_E - b_E\left(1-\tau^L\right)\left(1-l_t(\theta)\right)\theta w\right] dF(\theta) = \lambda Tax, \qquad (11)$$

$$\int_{\underline{\theta}}^{\theta_L^*} \left[a_L - b_L R_t s_{t-1}(\theta)\right] dF(\theta) = (1-\lambda)\, Tax, \qquad (12)$$

where the integrals on the left hand side of expressions (11) and (12) are over the fraction of the young respectively old population that have low enough ability endowment θ in order to be entitled to early, respectively late transfers. The government sets a mixture of parameters $\lambda, a_E, b_E, a_L, b_L, \tau^L$ and τ^K such that equations (11) and (12) hold.

2 Algorithm

Algorithm 1 *1. Discretize the space of innate abilities and form a vector* $\vec{\theta} = \left[\underline{\theta}, ..., \bar{\theta}\right]$, *so that an individual ability* $\theta_i \in \vec{\theta}$

2. Create a vector of population mass per ability level using the log normal distribution: $\vec{n} = \log normpdf\left(\vec{\theta}\right)$

3. Calculate total population size $N = \sum_\Theta \vec{n}$

4. Guess starting value for capital K

5. Start loop:

  (a) Derive factor prices $q, w,$ *and* R *using firm first order conditions*

  (b) Solve the household problem for each household i*:*

    i. solve for savings $s(\theta)$ *using* (2)

    ii. if $s < 0$, *set savings* $s = 0$

    iii. if $a_L - b_L Rs(\theta) < 0$ *solve again for savings using the equation for case:* $\theta \geq \theta_L^*$ *in expression* (3)

    iv. if $s < 0$, *set savings* $s = 0$

    v. calculate consumption c_t *and leisure* l_t

    vi. if $a_E - b_E\left(1-\tau^L\right) w\theta < 0$ *solve again for savings using the equation for case:* $\theta \geq \theta_E^*$ *in expression* (4)

    vii. if $l_t > 1$, *set* $l_t = 1$ *and solve again for savings using the maximization problem for* $l = 1$ *in* (8)

viii. *if* $s < 0$, *set savings* $s = 0$

ix. *if* $a_L - b_L Rs\left(T^{e*}\right) < 0$, *solve again for savings using the expressions for case:* $T^e \geq T^{e*}$ *in expression* (10)

x. *if* $s < 0$, *set savings* $s = 0$

xi. *calculate consumption when young* c_t

xii. *calculate consumption when old* c_{t+1}

(c) *Aggregate savings using population mass vector:*

$$K^{new} = S = \frac{1}{N}\sum\nolimits_{\Theta} s\left(\theta\right) \times \vec{n}$$

(d) *Clear government budget constraints* (11) *and* (12) *for* a_E *and for either:* a_L, b_L, τ^L, *or* τ^K

(e) *Calculate error:* $err = abs\left(K^{new} - K\right)$

(f) *Consistency check of aggregate resource constraint:* $C_{old} + C_{young} + S_{young} = Y + \left(1 - \delta\right)K$

(g) *if* $err >$ *tolerance, repeat from step (a) with* $K = 0.5K^{new} + 0.5K$

3 Sensitivity analysis

3.1 Changes in the intertemporal rate of substitution σ

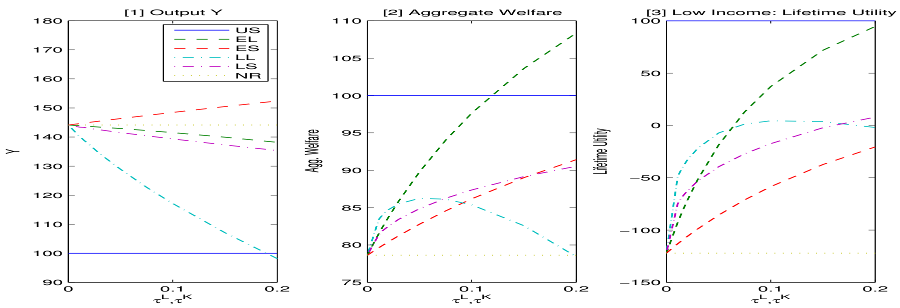

Figure 1: Sensitivity Analysis: $\sigma = 2.3$

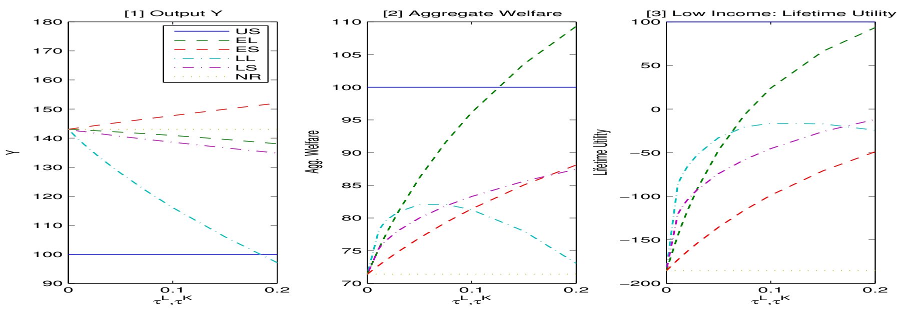

Figure 2: Sensitivity Analysis: $\sigma = 2.5$ (Benchmark)

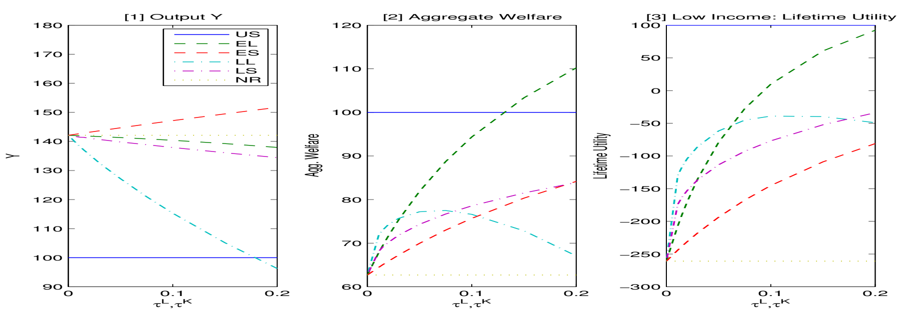

Figure 3: Sensitivity Analysis: $\sigma = 2.7$

3.2 Changes in the preference weight on consumption η

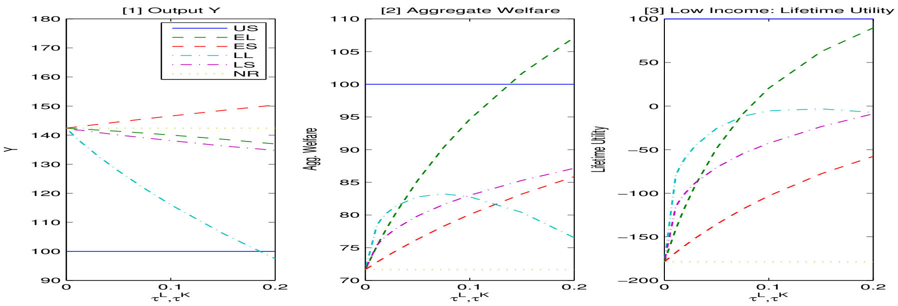

Figure 4: Sensitivity Analysis: $\eta = 0.34$

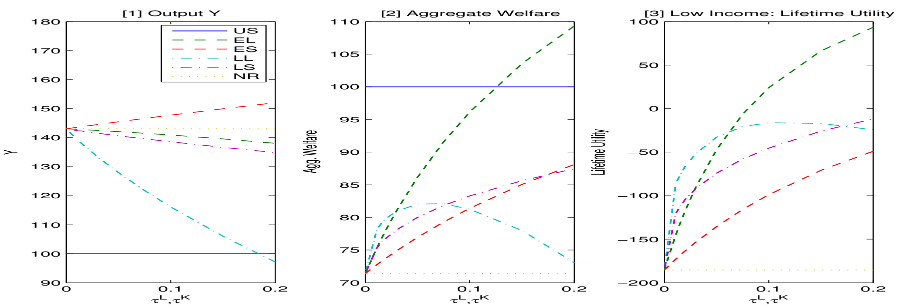

Figure 5: Sensitivity Analysis: $\eta = 0.37$ (Benchmark)

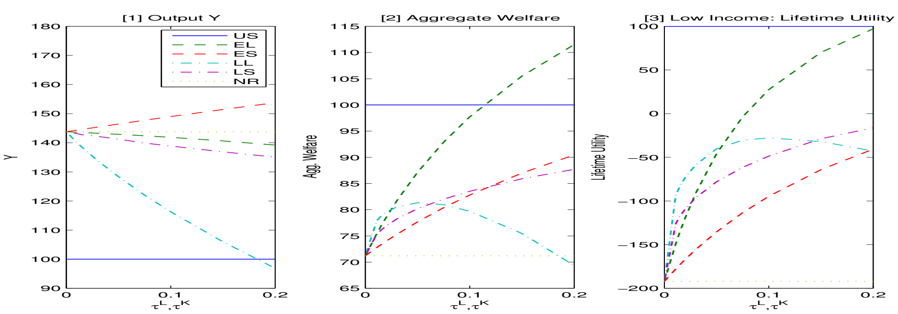

Figure 6: Sensitivity Analysis: $\eta = 0.40$

3.3 Changes in the lifetime income distribution parameter μ

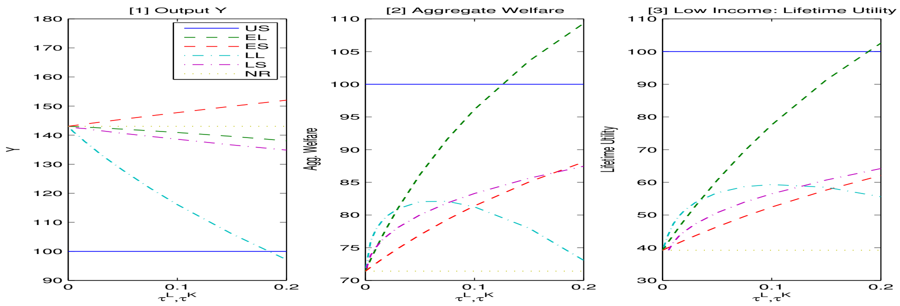

Figure 7: Sensitivity Analysis: $\mu = 13$, $(= US441,000)$

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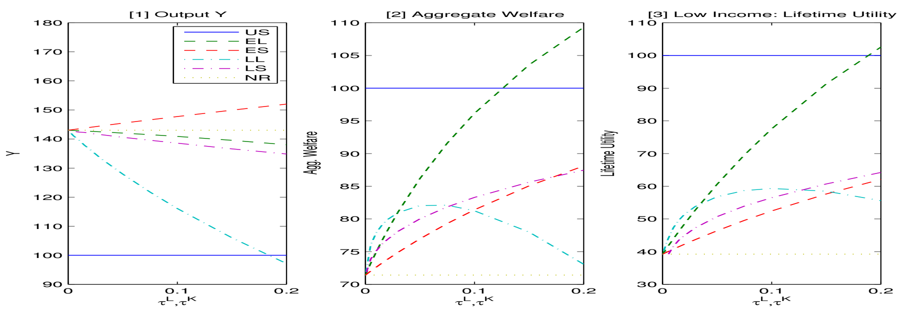

Figure 8: Sensitivity Analysis: $\mu = 13.4346$, $(= US683,000)$ Benchmark

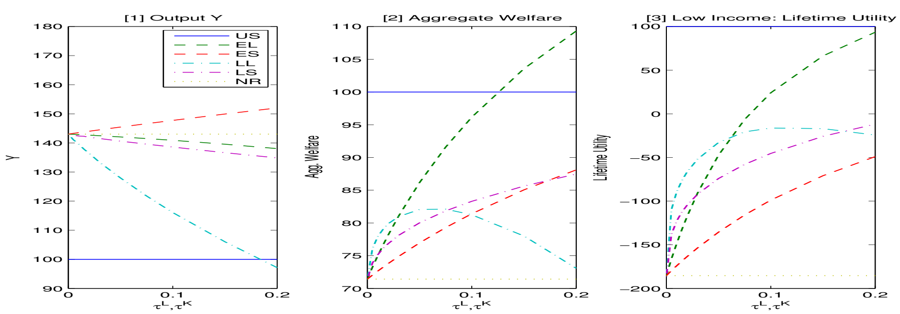

Figure 9: Sensitivity Analysis: $\mu = 13.6$, $(= US806,000)$

3.4 Changes in the lifetime income distribution parameter σ_I

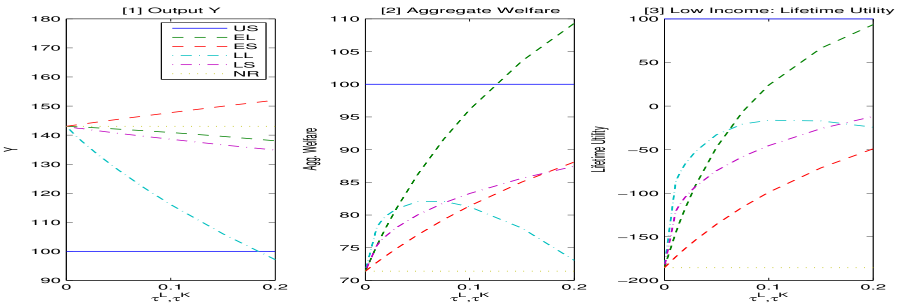

Figure 10: Sensitivity Analysis: $\sigma_I = 0.38$

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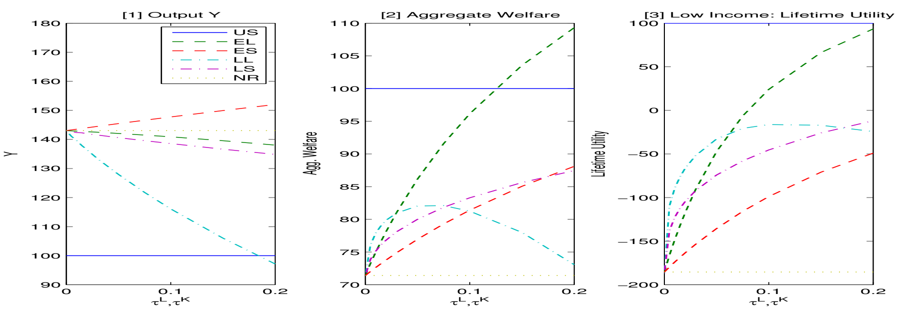

Figure 11: Sensitivity Analysis: $\sigma_I = 0.415$ (Benchmark)

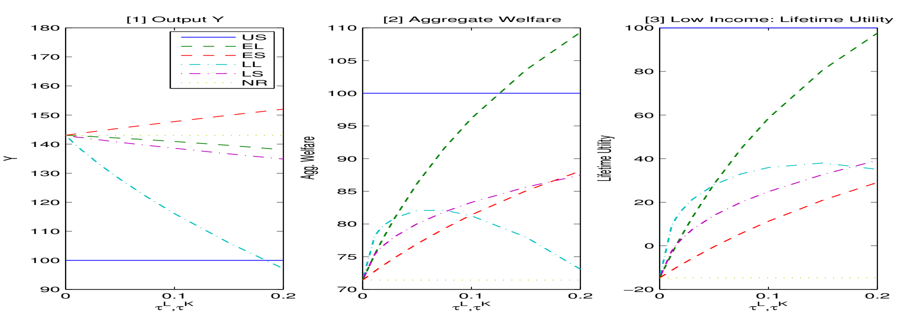

Figure 12: Sensitivity Analysis: $\sigma_I = 0.45$