The Timing of Redistribution*

Gerhard Glomm[†] Indiana University

Juergen Jung[‡] Towson University

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Abstract

We investigate whether late redistribution programs that can be targeted towards low income families, but may distort savings decisions, can "dominate" early redistribution programs that cannot be targeted due to information constraints. We use simple two-period OLG models with heterogeneous agents under six policy regimes: A model calibrated to the U.S. economy (benchmark), two early redistribution (lump sum) regimes, two (targeted) late redistribution regimes, and finally a model without taxes and redistribution. Redistribution programs are financed by a labor tax on the young and a capital tax on the old generation. We argue that late redistribution, if the programs are small in size, can dominate early redistribution in terms of welfare but not in terms of real output. Better targeting of low income households cannot completely offset savings distortions. In addition, we find that the optimal transfer and tax policy implies a capital tax of 100 percent and transfers exclusively to the young generation.

JEL Classification: H20, H22

Keywords: Taxation Timing, Transfer Timing, Capital Accumulation, Optimal Taxation, Capital Taxation

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†Corresponding Author: Gerhard Glomm, Department of Economics, Wylie Hall, Indiana University, Bloomington, IN, 47405, phone: (812) 855-7256, e-mail: gglomm@indiana.edu

‡Department of Economics, Towson University. Contact: jjung@towson.edu

1 Introduction

Most developed countries have large systems of transfers to the young, the middle aged and the old. In this paper we ask whether it is better to target these transfers to the young or to the old. We also study which generation should pay for these transfers. In his defense of the U.S. Social Security system Diamond (2004) gives a justification of why such transfers should occur late in the life-cycle. He points to imperfections in private annuity markets and workers' inability to insure against income and expenditure shocks as substantial advantages of the late redistribution scheme. A further advantage of late redistribution is the possibility of targeting transfers. Here we postulate that the government is more likely to observe the agent income type of an older agents as more information about this agent has become public. Of course, a natural consequence of late redistribution schemes as pointed out forcefully by Rust and Phelan (1997) and many others is the distortion of savings decisions.

Transfer schemes that redistribute early in the life-cycle do not have the same adverse distortionary effect on savings but tend to distort the labor-leisure decision. In the U.S. roughly 38 percent of all transfers to the adult population accrue to the $20-50$ year olds, with the remaining 62 percent going to the elderly. Programs that transfer resources predominantly to the $20-50$ year olds include food stamps, Medicaid, unemployment compensation, and Temporary Assistance for Needy Families (TANF). The two major programs that redistribute resources to the elderly are Social Security and Medicare.

In this paper we use an overlapping generations model where individuals live for two periods. Agents are heterogeneous with respect to their human capital (ability) endowment when they are born. Agents work when young and retire when old. Individuals face no uncertainty in our model. The model abstracts away any private intergenerational transfers. In this environment we ask the following two questions: (i) can late redistribution ever dominate early redistribution in terms of output and/or welfare, and (ii) what is the optimal mix of late vs. early redistribution and the respective tax rates that finance the transfers?

To answer the first question we compare an early redistribution regime to a late redistribution regime in terms of output and welfare measures. In addition, we impose an extreme assumption on the early redistribution program that clearly favors the late redistribution program. We impose that when the government transfers funds to young agents, the government is not able to observe the income type (ability type) of an agent, so that the government can only resort to (non-targeted) lump-sum transfers. In the second regime, the government redistributes to old agents only. In this case, we assume that the government is able to observe the income type and is therefore able to target transfers to the old agent. Since the targeting depends on savings, targeting itself becomes distortive. This will partly offset the benefit of targeting. We then compare the two regimes under varying financing options. Some of these regimes will allow for intergenerational transfers, others will exclude them.

In this scenario we find that late redistribution, although it introduces direct savings distortions into the model, can dominate early redistribution in terms of welfare. This result will depend crucially on the overall size of the redistribution program and on the level of targeting. It turns out that only for very small redistribution programs, late redistribution dominates early redistribution in terms of welfare only. Late redistribution cannot dominate early redistribution in terms of output (even though we made favorable assumptions towards the late redistribution regime). If the programs become larger, savings distortions offset the efficiency gains from targeting and welfare falls below the early redistribution levels. We show that the poor benefit from shifting redistribution away from the current regime benchmark case.

Finally, we calculate the policy that maximizes aggregate welfare. We find that relative to current U.S. policy, the optimal transfer and taxation policy suggests to transfer exclusively to the young generation and to increase capital taxes to 100 percent.

The literature on redistribution is tremendously rich and a lot of emphasis has been placed on efficient redistribution policies, optimal taxation and the public provision of education, unemployment benefits and retirement pensions. There is a large body of literature studying these redistribution programs.

Seshadri and Yuki (2004) study various forms of redistribution and their effect on the distributions of earnings and consumption. Studies analyzing the effects of redistribution programs on the labor market include Braeuniger (2004), Bhattacharya and Reed (2003), and Corneo and Marquardt (2000). Studies analyzing public vs. private funding of education and Social Security include Glomm and Kaganovich (2003), Gradstein and Kaganovich (2004), and Boldrin and Montes (2004). A number of papers is dedicated to risk sharing among generations under pay-as-you-go (PAYG) Social Security systems (e.g. Hassler and Lindbeck (1998)) or the value of information on production economies under uncertainty and its role on income inequality (e.g. Eckwert and Zilcha (2001) and Eckwert and Zilcha (2003)). None of these papers focuses on the timing of public redistribution programs and the theoretical informational advantage of late redistribution programs.

The classic contributions to the optimal tax literature are Mirrlees (1971) and Diamond and Mirrlees (1971). Kocherlakota (2005) surveys the literature on dynamic extensions of the original Mirrlees model. Important contributions to this literature are Chamley (1986) and Judd (1985) who find that capital should not be taxed in the steady state. This result also holds in many endogenous growth models as shown by Jones and Manuelli (1992) and Jones, Manuelli and Rossi (1997). In the Lucas (1990) model of human capital accumulation, however, Gruner and Heer (2000) show that the optimal flat-rate tax on capital income is well above zero. In addition, Hubbard and Judd (1986), Aiyagari (1995), and Imrohoroglu (1998) find that if households face tight borrowing constraints or cannot insure against idiosyncratic income shocks, then a positive capital tax cannot be ruled out in the optimum.

Early work on optimal taxation in models with households that have finite life time (i.e. Atkinson and Sandmo (1980), Auerbach, Kotlikoff and Skinner (1983)) already show that the optimal capital income tax is not zero in such models. Alvarez, Burbidge, Farrell and Palmer (1992), Garriga (2000), Erosa and Gervais (2002), and Conesa, Kitao and Krueger (2009) analyze this question in life cycle models and overlapping generations economies and conclude that a positive capital tax is optimal, partly caused by income profiles that increase over the life-cycle of an agent. A capital tax can then help redistribute from high income cohorts to lower income cohorts in the absence of a progressive labor income tax. Additional models that address optimal taxation in overlapping generations frameworks together with government commitment and information problems have been analyzed in Brett (1998), Blackorby and Brett (2000), and Pirttila and Tuomola (2001).

The plan for the rest of the paper is as follows. The next section outlines the model and defines equilibrium. Section 3 describes how we solve the model. Section 4 presents the calibration of the model to U.S. data. In section 5 we conduct policy analysis by changing the size of the distribution programs as well as the targeting levels of late redistribution programs when early redistribution is limited to lump-sum transfers. Section 6 discusses optimal tax policy. Section 7 concludes the paper. The appendix contains all tables and figures. A technical appendix is available upon request from the authors and contains the derivation of all solutions, a description of the algorithm that was used to numerically solve the model, and more detailed results discussed in the section on sensitivity analysis.

2 The Model

2.1 Demographics and Heterogeneity

We consider a two-period overlapping generations economy with heterogeneous agents. There is no population growth and the size of the population is normalized to one in each period. Agents differ with respect to their individual ability, denoted θ^i. We assume that θ^i is an iid random variable that is distributed according to a time invariant distribution function F with support in $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$, where $\underline{\theta}, \overline{\theta} \in \mathbb{R}_+$, and i indexes all agents. We will drop the i superscript in order to not clutter the notation. An agent's effective unit of labor depends on ability level θ. For simplicity we assume the identity function to describe the relation between innate ability and effective unit of labor, so that we use θ interchangeably for ability or income.

Agents are endowed with one unit of time that they can either consume as leisure l_t or supply as labor $(1 - l_t)$ earning wages. The effective human capital per individual that enters the production process is

$$h_t(\theta) = (1 - l_t)\,\theta.$$

2.2 Preferences and Technology

Preferences of an individual agent in generation t are given by the utility function

$$u(c_t, c_{t+1}, l_t) = \frac{\left(c_t^{\eta} l_t^{1-\eta}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma}, \tag{1}$$

where c_t and c_{t+1} are consumption when young and old, l_t is leisure, η is the preference weight on consumption, $1-\eta$ is the preference weight on leisure, σ is then inverse of the intertemporal elasticity of substitution, and β is the time preference factor.

The economy's aggregate production function is

$$Y_t = A K_t^{\alpha} H_t^{1-\alpha}, \tag{2}$$

where parameters $A > 0$, $0 < \alpha < 1$, Y_t is total output, K_t is the aggregate capital stock of physical capital, and H_t is the aggregate capital stock of human capital in period t. Physical capital K_t will be financed by aggregate savings of the previous generation S_{t-1} and depreciates each period at rate δ.

2.3 Government

The government collects taxes and gives transfers to young and old agents. The government cannot issue debt and has to balance its budget every period. Government consumption is set equal to zero. The government collects a labor tax τ^L from the young generation and a capital tax τ^K on the interest of savings income from the old generation. Total tax revenue can be expressed as

$$Tax_t = \tau^L \int_{\Theta} w_t h_t(\theta)\, dF(\theta) + \tau^K \int_{\Theta} r_t s_{t-1}(\theta)\, dF(\theta), \tag{3}$$

where r_t is the interest rate on capital.

In the following we will distinguish two government redistribution programs that are financed with tax revenue. The two programs will differ with respect to when redistribution takes place in an agent's life. The first program gives targeted transfers to young agents. The size of the transfers depends on the agents' wage income. The second program gives targeted transfers to the old generation. The size of these transfers depends on the individuals' savings income. The targeting function for early transfers can be written as

$$T^{E}(\theta) = \max\left[0,\ a_E - b_E\left(1-\tau^{L}\right) w_t h_t(\theta)\right], \tag{4}$$

where $\left(1-\tau^{L}\right) w_t h_t(\theta)$ is the after tax labor income in the first period, $0 \leq a_E$ represents the maximum transfer, and $0 \leq b_E$ captures the degree of means-testing of the transfer program. As labor income increases, the transfers decreases at rate b_E.

The government uses an exogenous fraction λ of its total tax income over the length of a period (approximately thirty years in this OLG setting) to pay for this early redistribution program. The government budget constraint for the early transfer case is

$$T^{E} = \int_{\Theta} \max\left[0,\ a_E - b_E\left(1-\tau^{L}\right) w_t h_t(\theta)\right] dF(\theta) = \lambda \times Tax_t. \tag{5}$$

The second program gives transfers exclusively to old agents based on the agents' wealth. The lower the wealth level, the more transfers an agent will receive from the government. The targeting formula for the late redistribution program is

$$T^{L}(\theta) = \max\left[0,\ a_L - b_L R_t s_{t-1}(\theta)\right], \tag{6}$$

where $R_t s_{t-1}(\theta)$ is wealth (from savings plus interest net of capital taxes) in the second period, $0 \leq a_L$ represents the maximum transfer, and $0 \leq b_L$ captures the degree of means-testing of the transfer program. As wealth $R_t s_{t-1}(\theta)$ increases, transfers decrease at rate b_L. The government uses the residual tax income, that is fraction $(1-\lambda)$ of total tax income, to finance this program. The government budget constraint for the late transfer program can be written as

$$T^{L} = \int_{\Theta} \max\left[0,\ a_L - b_L R_t s_{t-1}(\theta)\right] dF(\theta) = (1-\lambda) \times Tax_t. \tag{7}$$

2.4 Households

Agents know the government policy and the transfer functions. In addition they are borrowing constrained. They maximize

$$\max_{\{c_t,\ l_t,\ c_{t+1},\ s_t\}} \left\{ \frac{\left(c_t^{\eta} l_t^{1-\eta}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right\},\ \text{s.t.} \tag{8}$$

$$c_t + s_t = \left(1-\tau^{L}\right)(1-l_t)\,\theta w_t + T^{E}\left[\left(1-\tau^{L}\right)(1-l_t)\,\theta w_t\right], \tag{9}$$

$$c_{t+1} = \left(1+\left(1-\tau^{K}\right) r_{t+1}\right) s_t + T^{L}\left[\left(1+\left(1-\tau^{K}\right) r_{t+1}\right) s_t\right], \tag{10}$$

$$0 \leq l_t \leq 1,\ s_t \geq 0, \tag{11}$$

where c_t and c_{t+1} are consumption when young and old, l_t is leisure when young, s_t is savings, θ is the efficiency unit of labor or ability, so that $(1-l_t)\,\theta$ becomes effectively supplied human capital, which we denote by $h_t(\theta)$, w_t is the wage rate, r_{t+1} is the interest rate, τ^{L} is the labor tax rate, τ^{K} is the capital tax rate, and T^{E} and T^{L} are the early and late transfer functions, respectively. All household choices are functions of the exogenous realization of innate ability θ.

2.5 Firms

We assume a representative firm that uses a standard Cobb-Douglas technology. The firm solves

$$\max_{\{K_t,\ H_t\}} \left\{ AK_t^{\alpha} H_t^{1-\alpha} - q_t K_t - w_t H_t \right\}, \tag{12}$$

taking prices (q_t, w_t) as given.

2.6 Equilibrium

Definition 1 *A competitive equilibrium is a collection of sequences of distributions of individual household decisions* $\{c_t(\theta), c_{t+1}(\theta), l_t(\theta), s_t(\theta)\}_{t=0}^{\infty}$ *for all agents* $\theta \in \Theta$*, sequences of aggregate stocks of physical and human capital* $\{K_t, H_t\}_{t=0}^{\infty}$*, sequences of factor prices* $\{w_t, q_t, r_t, R_t\}_{t=0}^{\infty}$*, sequences of government expenditures* $\left\{T_t^E, T_t^L\right\}_{t=0}^{\infty}$ *and government policy parameters* $\left\{a_E, b_E, a_L, b_L, \tau^L, \tau^K, \lambda\right\}$ *such that*

(i) the sequence $\{c_t(\theta), c_{t+1}(\theta), l_t(\theta), s_t(\theta)\}_{t=0}^{\infty}$ *solves the maximization problem of the household* (8) *for each agent* $\theta \in \Theta$,

(ii) factor prices are determined by

$$q_t = \alpha Y_t / K_t = \alpha A K_t^{\alpha-1} H_t^{1-\alpha}, \tag{13}$$

$$w_t = (1-\alpha) Y_t / H_t = (1-\alpha) A K_t^{\alpha} H_t^{-\alpha}, \tag{14}$$

$$R_t = 1 + \left(1-\tau^K\right)(q_t - \delta) \equiv 1 + \left(1-\tau^K\right) r_t, \tag{15}$$

(iii) capital markets clear, so that aggregate capital stocks are given by

$$K_{t+1} = S_t = \int_{\Theta} s_t(\theta)\, dF(\theta),$$

$$H_t = \int_{\Theta} ((1 - l_t(\theta)) \times \theta)\, dF(\theta),$$

(iv) commodity markets clear

$$C_{old,t} + C_{young,t} + S_{young,t} = Y_t + (1-\delta) K_t, \tag{16}$$

(v) and the respective government budget constraints (5) *and* (7) *hold.*

3 Solving the Model

The model is solved for steady states. We will therefore remove all time subscripts from here onwards.

3.1 Households

Before substituting the budget constraints we first find the optimal relation between consumption and leisure. The ratio of marginal utilities has to equal the price ratio when solutions for leisure are interior, i.e. for $l \in (0, 1)$ it holds that

$$\frac{(1-\eta)c}{\eta l} = (1 - b_E)\left(1 - \tau^L\right) w\theta.$$

We can now express leisure in terms of consumption as

$$l_t = \min\left(\Theta\left(\theta\right) c, 1\right), \tag{17}$$

where $\Theta\left(\theta\right) = \frac{((1-\eta)/\eta)}{(1-b_E)(1-\tau^L)w\theta}$. Leisure is restricted to be within zero and one. This substitution changes the budget constraint in the first period to

$$p_c c + s = (1 - b_E)\left(1 - \tau^L\right) w\theta + a_E,$$

where $p_c = \left(1 + \frac{1-\eta}{\eta}\right)$.[1]

In addition, an individual agent's maximization problem depends on whether she receives transfers or not. This will of course depend on the agent's initial ability level which in turn determines her income. We will have to calculate threshold ability levels θ_E^* and θ_L^* that determine whether an agent is poor enough to receive means tested early or late transfers, respectively. Whenever the initial ability/income level is below the threshold $\theta < \theta_E^*$, then the agent's labor income is low enough in order for early transfers to be positive, $T^E\left(\theta\right) > 0$. Whenever the agent's innate ability is equal or above this threshold, $\theta \geq \theta_E^*$ then early transfers will be zero, $T^E\left(\theta\right) = 0$. Similar conditions on ability hold for transfers in the second period. When second period wealth is low enough based on the agent's ability $\theta < \theta_L^*$, then late transfers are positive, $T^L\left(\theta\right) > 0$. Whenever the initial ability is equal or above this threshold, $\theta \geq \theta_L^*$, then late transfers will be zero, $T^L\left(\theta\right) = 0$. This will generate four agent types, type one receives transfers when young and old, type two receives transfers when young only, type three receives transfers when old only, and type four does not receive any transfers.

The solution will result in two cases, one with interior solutions for leisure and the second one with a corner solution for leisure. The policy functions for optimal savings for the interior

[1] This applies for the interior solution case. See the Technical Appendix that is available upon request from the authors for details of this derivation.

solution case are

$$\theta < \theta_E^* \wedge \theta < \theta_L^* : s(\theta) = \frac{\left[(1-b_E)\left(1-\tau^L\right)\theta w + a_E\right]\left[\frac{p_c\beta(1-b_L)R}{\chi(\theta)}\right]^{\frac{1}{\sigma}} - p_c a_L}{p_c(1-b_L)R + \left[\frac{p_c\beta(1-b_L)R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}, \tag{18}$$

$$\theta < \theta_E^* \wedge \theta \geq \theta_L^* : s(\theta) = \frac{\left[(1-b_E)\left(1-\tau^L\right)\theta w + a_E\right]\left[\frac{p_c\beta R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}{p_c R + \left[\frac{p_c\beta R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}, \tag{19}$$

$$\theta \geq \theta_E^* \wedge \theta < \theta_L^* : s(\theta) = \frac{\left(1-\tau^L\right)\theta w\left[\frac{p_c\beta(1-b_L)R}{\chi(\theta)}\right]^{\frac{1}{\sigma}} - p_c a_L}{p_c(1-b_L)R + \left[\frac{p_c\beta(1-b_L)R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}, \tag{20}$$

$$\theta \geq \theta_E^* \wedge \theta \geq \theta_L^* : s(\theta) = \frac{\left(1-\tau^L\right)\theta w\left[\frac{p_c\beta R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}{p_c R + \left[\frac{p_c\beta R}{\chi(\theta)}\right]^{\frac{1}{\sigma}}}, \tag{21}$$

where $\chi(\theta) = \Theta(\theta)^{(1-\eta)(1-\sigma)}$.

For the corner solution case where leisure is equal to one the optimal savings functions can only be expressed as implicit functions of the following form

$$a_E < a_E^* : F(s) \equiv \eta(a_E - s)^{\eta(1-\sigma)-1} - \frac{\beta(1-b_L)R}{((1-b_L)Rs + a_L)^{\sigma}} = 0, \tag{22}$$

$$a_E \geq a_E^* : F(s) \equiv \eta(a_E - s)^{\eta(1-\sigma)-1} - \beta R(Rs)^{-\sigma} = 0. \tag{23}$$

In this case individual labor income is zero and first period income is therefore only a function of the fixed government transfer a_E. The level of a_E then determines whether agents will receive government transfers in their second period, so that the second period transfer parameters a_L and b_L come into play when determining the optimal savings rate s. Solutions to this case are numerical in nature. We can then solve for the threshold transfer level a_E^* that determines whether the old agent receives a targeted transfer. The criterion is again derived from the payout formula for late transfers, expression $T^L(\theta)$ and can be written as implicit function

$$G(a_E^*) \equiv a_L - b_L R\hat{s}(a_E) = 0,$$

where the optimal savings function $\hat{s}$ is now a function of the early lump-sum transfer level a_E as can be seen from expressions (22) and (23).

3.2 Government

Using the first order conditions from the firm's problem we can simplify the expression for total tax revenues (3) to

$$Tax = \left[\tau^L(1-\alpha) + \tau^K\alpha\right]AK^{\alpha}H^{1-\alpha} - \tau^K\delta K,$$

and the two government budget constraints reduce to

$$\int_{\underline{\theta}}^{\theta_E^*} \left[a_E - b_E\left(1-\tau^L\right)(1-l(\theta))\,\theta w\right] dF(\theta) = \lambda Tax, \tag{24}$$

$$\int_{\underline{\theta}}^{\theta_L^*} \left[a_L - b_L Rs(\theta)\right] dF(\theta) = (1-\lambda)\,Tax, \tag{25}$$

where the integrals on the left hand side of expressions (24) and (25) , respectively are over the fraction of the young and old populations that have low enough ability endowment θ in order to be entitled to transfers. The government sets a mixture of parameters $\lambda, a_E, b_E, a_L, b_L, \tau^L$ and τ^K such that equations (24) and (25) hold. Since we cannot get analytical solutions we use a variant of the Gauss-Seidl algorithm to solve the model numerically.[2]

4 Calibration

We present a summary of all parameter selections in table 2. We next describe how we chose the parameters for the benchmark calibration. In order to get a good idea about the parameter ranges and especially about the distribution of the unobserved ability parameter θ, we attempt to calibrate the six key moments that the model generates to U.S. data. A summary of these model generated moments and their U.S. data equivalent are presented in table 3.

4.1 Demographics and Heterogeneity

We use data on the lifetime income distribution reported in Fullerton and Rogers (1993) to calibrate the ability or skill distribution $F(\theta)$. Fullerton and Rogers (1993) use data from the Panel Study of Income Dynamics for the years 1970-87 and calculate mean lifetime income before and after taxes/transfers for each decile. We use these deciles of the after tax/transfers to get point estimates for the mean and standard deviation of a log normal distribution.[3]

We use an iterative procedure to estimate the parameters for the mean and the standard deviation of this distribution. We first draw $500,000$ log normally distributed random numbers. We then calculate the deciles and compare them to the deciles reported in in Fullerton and Rogers (1993). We then minimize the absolute distance of the simulated deciles and the deciles in the table by adjusting the appropriate mean and standard deviation parameters μ and σ_I. Point estimates for parameters μ and σ_I are reported in the third panel of table 2. To check the sensitivity of our results with respect to the functional form of our income distribution, we also fit a gamma distribution to the mean income data per decile. We report both estimated lifetime income distributions in the top panel of figure 1. The bottom panel plots the lifetime income figures per decile of our estimates against the estimates from Fullerton and Rogers (1993).

The estimated log normal distribution represents the lifetime income distribution. However, for our model we need the distribution of innate ability θ. We first normalize wages w to one by picking the appropriate total factor productivity A. The term $wh(\theta)$ is then equal to $h(\theta)$ and

[2] More details on the derivation of the policy functions and the numerical solution method are available in a technical appendix upon request from the authors.

[3] Fernandez and Rogerson (2003) use the same data for calibrating their model of education finance systems. They find that the pretax lifetime income distribution is very similar to the post tax and transfers distribution of lifetime income.

represents wage earnings over the 30 years of active work life, or period one in our model.[4] We use the log normal distribution as our benchmark model. It is important to keep in mind that our estimated log normal distribution is the distribution of lifetime income and not of lifetime earnings. We therefore conduct sensitivity analysis on parameters μ and σ_I in section 5.3. We calibrate the lowest lifetime income individual as $\underline{\theta} = \$1,000$ and the highest lifetime income individual at $\bar{\theta} = \$5,000,000$ which is well above the mean lifetime income of \$1.7 million for the $98 - 100$ percentile in Fullerton and Rogers (1993).[5]

4.2 Preferences and Technology

We pick the total factor productivity A to normalize wages to one. The capital share of production $\alpha = 0.36$ as in Kydland and Prescott (1982). The annual depreciation rate is $\delta = 8$ percent which falls in between the estimates in Nadiri and Prucha (1996) who report numbers between $5.9 - 12$ percent. The time preference rate is $\beta = 0.953$ and the inverse of the intertemporal rate of substitution is $\sigma = 2.5$. Parameter β and σ together are set to match the capital output ratio and the annual interest rate to standard values that can be found in the NIPA accounts.[6]

We chose $1 - \eta = 0.63$ (the share on leisure) so that average lifetime labor supply equals 0.37 which is close to 0.374 as estimated by Gomes, Kotlikoff and Viceira (2007). The coefficient of relative risk aversion in consumption is then given by $-\frac{cu_{cc}}{u_c} = \sigma\eta + 1 - \eta = 1.57$, so that the intertemporal elasticity of substitution is 0.64. We summarize the calibration parameters in table 2.[7]

4.3 Government

We next calibrate the shares of labor tax revenue and capital tax revenue from federal, state and local tax revenues. Table 5 contains data on tax revenue from the U.S. Census and the IRS of fiscal year 2004. Table 6 translates these revenues into labor tax revenues and capital tax revenues in the model. We disregard consumption and sales tax revenues and other government income, since they are not part of our model. We find that 75 percent of tax income comes from labor taxes and 25 percent comes from capital taxes. Total tax revenue from labor and capital taxes amounts to 21.4 percent of GDP. We target this fraction of GDP to be the size of government in our model. The corresponding labor tax rate turns out to be $\tau^L = 27$ percent and the capital tax rate is $\tau^K = 17$ percent.

Table 7 contains data on government spending of fiscal year 2004. We use these data to calibrate the relative share of total tax revenue going into early vs. late transfers as governed by parameter λ. In table 1 we classify government transfers into transfers to the young population aged $20 - 50$ and transfers to the old population aged $50 - 80$. We conclude that 38 percent of all government transfers to adults (age $20 - 80$) goes to the young population (age $20 - 50$), whereas the residual 62 percent goes to the old (age $50 - 80$). Hence we set $\lambda = 0.38$ in the benchmark economy.

[4] In our model $h(\theta) = (1 - l(\theta))\theta$. From the household's problem l does depend on θ. So that h is a function of the type θ. The data pins down the distribution of h since w is normalized to one. If $h(\theta)$ is monotonic it can be inverted and then the distribution of h, i.e. the data, pins down the distribution of θ.

[5] The actual distribution that we use here is a truncated log normal distribution. The truncation is required for computational/numerical reasons and does not affect the results of the paper as the the agent mass outside of the truncation is close to zero.

[6] It is clear that in a general equilibrium model every parameter affects all equilibrium variables. Here we associate parameters with those equilibrium variables that are the most quantitatively affected.

[7] We also find that our results are robust to small changes in the values of σ and η. These results of this sensitivity analysis are presented in section 5.3.

The main contributing programs to the old are Social Security ($496 Billion) and Medicare ($296 Billion). When splitting Medicaid into transfers to young and old we use information from Kaiser (2005) and allocate 31 percent of Medicaid expenditures to the old generation. We then split the category"Health other"in table 1 in a similar fashion. The split for housing assistance is provided in Kochera (2001) (66.6 percent to the young generation) and the split of the food-stamp program is detailed in Kassner (2001) (91 percent to the young generation). Spending on primary and secondary education is excluded because we only model the population from age 20 upwards.

For the benchmark targeting program we choose $a_E = 0.19$ (˜$346,000) and $a_L = 0.22$ (˜$361,000) as the fixed portion of the early and late transfer schemes. This translates into a maximum monthly transfer of roughly $1,000 for early and late transfers. The parameter that governs the targeting rate is set to $b_E = 0.3$ and the parameter for late targeting is $b_L = 0.2$. In general early transfer programs are more targeted (e.g. compare the stipulations for Medicaid, foodstamps, etc. vs. Social Security and Medicare).

There are multiple pairs of (a_E, b_E) and (a_L, b_L) that would satisfy the respective government budget requirement of expressions (5) and (7) respectively. We think this is a good parameterization as young people without any labor income will receive roughly $1,000 and old people without any savings income will receive a $1,000 monthly benefit. According to Olsen and Hoffmeyer (2002) the special minimum benefit out of Social Security amounts to $500 on average per month (as of February 2002). If one factors in that this number is about $2,000 below the annual poverty income level, we think it is reasonable that individuals with zero savings get an amount larger than the minimum social security benefit. In addition, our late redistribution program is not only Social Security but includes other transfers to the old, like Medicare, as well. The current parameterization ensures that about 77 percent of young individuals receive some form of early transfers and 97 percent of old individuals receive late transfers.

While the 97 percent coverage rate for the old in the model is a reasonable approximation for the almost universal coverage in the US for the elderly through Social Security and Medicare, the model's 77 percent coverage rate for the young may at first appear to be high. We would like to point out that transfers to the young include food stamps, housing subsidies, unemployment payments, Medicaid payments and public subsidies for higher education (i.e. FAFSA). Currently there are about 20 million students attending institutions of higher education and about 14 million, or 70 percent receive FAFSA assistance. Putting all these programs together may well generate coverage rates consistent with the model.

5 Policy Analysis of Different Tax Policy Programs

In this section we analyze whether an early redistribution program can ever be dominated by a late redistribution program in terms of output and/or welfare. Since a transfer to old agents causes strong negative effects on capital accumulation in OLG models, the late redistribution program is easily dominated by early redistribution. So in order to find potential cases where late redistribution can actually outperform early redistribution we need to take an extreme stance and assume initially that only late redistribution can be targeted.

In the following we therefore set $a_E > 0$ and force $b_E = 0$. We conduct our investigation using six separate policy specifications: (i) the U.S. economy denoted US with the current split of transfers between young and old as described in the calibration section functions as a benchmark, (ii) early redistribution to the young generation financed by taxes on wage income of the current young generation denoted EL, (iii) early redistribution to the young generation financed by taxes on interest income of the current old generation denoted ES, (iv) late redistribution to the old generation financed by taxes on wage income of the current young generation denoted LL, (v) late redistribution to the old generation financed by taxes on interest income of the current old generation denoted LS, and (vi) no redistribution denoted NR.[8] We summarize the parameter settings for these five cases as:

$$\begin{aligned}
\text{United States } (US) &: \tau^L = 27\%,\ \tau^K = 17\%,\ \lambda = 0.38, \\
\text{Early transfer with labor tax } (EL) &: \tau^L > 0,\ \tau^K = 0,\ \lambda = 1, \\
\text{Early transfer with capital tax } (ES) &: \tau^L = 0,\ \tau^K > 0,\ \lambda = 1, \\
\text{Late transfer with labor tax } (LL) &: \tau^L > 0,\ \tau^K = 0,\ \lambda = 0, \\
\text{Late transfer with capital tax } (LS) &: \tau^L = 0,\ \tau^K > 0,\ \lambda = 0, \\
\text{No transfer, no taxes } (NR) &: \tau^L = \tau^K = 0.
\end{aligned}$$

5.1 Size of Redistribution Program

Our first experiment is to find out which of the two programs, early or late redistribution, is better in terms of output and welfare. We therefore compare the different regimes adjusting the tax rates that finance them. In the case of regimes EL and LL we choose labor taxes in the range between $0-20$ percent and for regimes ES and LS we use capital tax rates in the same range from $0-20$ percent. These tax rates are the only source of funding for the redistribution programs in the respective regimes and therefore directly determine the size of these programs.

Output. Figures 2 to 5 present the results. In order to relate the four distribution regimes EL, LL, ES, and LS to our benchmark calibration we also plot the original calibration of the U.S. economy denoted US as well as the no redistribution regime with zero taxes NR. For the late redistribution regimes we choose targeting parameter $b_L = 0.2$ and let the second targeting parameter a_L adjust to clear the government budget constraint as we alter the tax rates. Technology and agent heterogeneity is identical for all regimes. Note that the size of the economies of the different regimes are not equal and our experiments are therefore not revenue neutral. Our goal is simply to find ranges for tax rates where late redistribution programs outperform early redistribution programs in terms of output and welfare.

Figure 2 presents the aggregate economy. In panel [1] we see that aggregate output of early redistribution financed with a labor tax, EL is almost identical to the no redistribution case NR. The slight difference between the two cases is explained by the labor distortion caused by the labor tax.[9] Early redistribution financed by a savings tax on the old generation, ES,

[8] Borrowing constraints could potentially affect the 2 policy regimes with exclusive late redistribution LL and LS - as low income agents could be tempted to borrow against future late transfers. However, this is only the case for an extremely small number of agents (i.e. close to zero percent of the population!). Solving the model without borrowing constraints would therefore not change any of the results.

[9] It can be shown that with inelastic labor supply the two cases, EL and NR will be identical as tax revenues collected from the young are returned to them immediately in a lump sum fashion so that aggregate savings will be identical. The only difference then is in terms of welfare as the lump sum tax provides some redistribution from the rich to the poor.

produces the largest steady state output (red dotted line). We see from panel [2] that in this case aggregate savings is the highest and therefore the level of steady state output dominates the other redistribution regimes. This is true for all sizes of government, $\tau = 0$ to $\tau = 20$ percent. The only program that "grows" the economy monotonically in terms of output while increasing the tax rates is the ES program (see panel 1). Here the additional funds collected via the capital tax are shifted to the young. As labor is not taxed, the distortion of labor supply caused by raising the capital tax on savings, is relatively small and the larger stock in physical capital ensures that output increases. The early redistribution case financed by a labor tax also exhibits an increase in the savings rate of physical capital. However, the increase in the labor tax rate causes a strong decrease in labor supply that more than offsets the increase in physical capital. Therefore output for the EL (green dotted line) drops as the redistribution program becomes larger.

The volume of redistribution VR, or government size, is presented in panel [3] of figure 2 We define VR as the total amount of funds collected by the government in the steady state which can be written as

$$
\begin{aligned}
US &: \; VR = \int_{\Theta} \left[\tau^{L} wh\left(\theta\right) + \tau^{K}\left(q-\delta\right)s\left(\theta\right)\right] dF\left(\theta\right) = \left[\tau^{L}\left(1-\alpha\right) + \tau^{K}\alpha\right] Y - \tau^{K}\delta K, \\
EL, LL &: \; VR = \int_{\Theta} \tau^{L} wh\left(\theta\right) dF\left(\theta\right) = \tau^{L}\left(1-\alpha\right) Y, \\
ES, LS &: \; VR = \int_{\Theta} \tau^{K}\left(q-\delta\right) s\left(\theta\right) dF\left(\theta\right) = \tau^{K}\alpha Y - \tau^{K}\delta K, \\
NR &: \; VR = 0,
\end{aligned}
$$

for the six cases considered. We see from panel [3] that the early redistribution with taxes on labor, EL, generates the largest volume of redistribution in absolute terms. Keep in mind that the size of government as a fraction of GDP is identical to $\tau^{L}\left(1-\alpha\right)$ for the regimes using the labor tax (EL, LL) and equal to $\tau^{K}\left(\alpha Y - \delta K\right)$ for regimes using the capital tax (ES, LS). It is now easy to see that for identical tax rates $\tau^{L} = \tau^{K}$ the labor tax financed programs (EL, LL) will always have a larger government as percentage of GDP since $\left(1-\alpha\right) Y > \left(\alpha Y - \delta K\right)$ given our calibration. This translates into larger redistribution volumes (VR) in levels for EL and LL as well, despite the fact that the early redistribution program with capital taxes, ES, produces the largest economy.

The results of aggregate labor presented in panel [4] of figure 2 have a straightforward interpretation. Labor supply is lowest for the regimes that finance redistribution with a labor tax. The labor tax directly decreases the price of leisure and we see a decrease in the labor supply. In addition we observe that late redistribution programs cause households to save less. This behavior leads to lower levels of physical capital in late redistribution programs. Consequently, in equilibrium interest rates increase. Panels [5] and [6] plot the interest rate and the wage rates respectively. Interest rates are mostly decreasing in tax rates, except for the LL case. In this case the labor tax and the late transfer decrease savings which lowers the capital stock and increases the interest rate. In general, late redistribution regimes produce smaller wage rates and larger interest rates.[10]

From the first figure we conclude that early redistribution programs dominate late redis-

[10] In these experiments, parameter a_L adjusts endogenously to exogenous changes in taxes whereas b_L is held fixed at the original 0.2. As we increase the tax rate in the late redistribution regimes the lump sum transfers a_L increase in order to keep the government budget balanced. This results in more generous late transfer schemes. As taxes increase the size of early redistribution programs increases as well in order to clear the budget constraint of the government. These results are also depicted in panel [3] and have been discussed earlier.

tribution programs in terms of output. Early transfers work as an "engine of growth" and the larger the early redistribution program is, the larger stock of physical capital becomes. If distortions in the labor market do not completely offset these increases in physical capital accumulation, the entire economy will grow (see regime ES, red dotted line). Similar results for OLG models have been reported in Jones and Manuelli (1992). OLG models have the feature that the young generation has to buy the entire capital stock from the old generation. If income of the young generation is too low, then the young cannot afford to buy an ever increasing capital stock and the economy will not grow. Jones and Manuelli (1992) find that income taxes (even taxes on capital) that can be used to finance transfers to the young generation, will allow the young to buy an ever increasing capital stock and economic growth is possible. We abstract from growth in our model, but a similar mechanism ensures that early redistribution programs generate larger steady state income.

Welfare. We next turn our attention to welfare analysis and investigate whether late redistribution programs can dominate early redistribution programs in terms of welfare. We first define lifetime utility of an agent type θ as

$$U(\theta) = u\left(c^{y}(\theta), l(\theta)\right) + \beta u\left(c^{o}(\theta)\right). \tag{26}$$

Aggregate welfare is then defined as the aggregate lifetime utilities of all individuals born directly into the steady state. Aggregate welfare therefore is

$$W = \int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\, dF(\theta). \tag{27}$$

Figure 3 presents aggregate welfare in panel [1] and lifetime utility of a low income individual (lifetime income of US $250,000) in panel [2] . From panel [1] we see that aggregate welfare is an increasing function of the size of the redistribution program for the early transfer regimes EL and ES. This again reflects the growth effects of early transfers in OLG models. The aggregate welfare of the late redistribution regime is non-monotonic in tax rates. For very small transfer programs the savings distortions are very low and the redistributive effects of the targeted late redistribution program will dominate. Aggregate welfare levels of late redistribution programs also dominate those of their early redistribution counterparts. However, this is only the case for programs that can be financed with a flat tax rate on labor or capital smaller than 3 percent. For late redistribution programs larger than that savings distortions become too strong and aggregate welfare levels begin to drop. We see that poor individuals benefit the most from targeted late redistribution if the program is kept very small.

Finally, figure 4 contrasts lifetime utility levels of individuals who represent the 25^{th}, 50^{th}, 75^{th}, and the 100^{th} life time income percentile (panels $[1]-[4]$). As one would expect, the low income groups tend to benefit from larger redistribution programs, whereas high income groups lose. The program that does worst in terms of welfare is the late redistribution case financed via a labor tax, LL. This regime exhibits a dual distortion. The savings distortion from late transfers is augmented with labor supply distortions from taxing labor. Therefore welfare for almost all income groups is decreasing in the size of the redistribution program.[11] Regime ES (red dotted line) increases welfare of almost all groups as the size of the program becomes larger. Even the highest quartile experiences some welfare improvement over the no tax (NR) case. One reason is the large growth effect of this program. We conclude that late redistribution programs can dominate early redistribution programs in terms of welfare only

[11] The only exception is the very low income group as reported in panel [1] of figure 4.

when the size of the redistribution is kept very small.

We do not observe the domination of LL for small programs in figure 4 (where we depict the 25^{th}, 50^{th}, 75^{th}, and the 100^{th} life time income types), but we see it in figure 3 (the low income type in panel [2] but also at the aggregate level where we add all agents equally weighted in panel [1]). The fact that the low income type benefits from the targeted late redistribution reform when the program is small is not surprising (panel [2] in figure 3). The targeting plus the small size will not result in large adverse growth effects but directly benefit the relatively poor agent). The early redistribution program that is not targeted on the other hand creates larger distortions and benefits the low income types less strongly (since it is not targeted). The fact that we still see this dominance of LL at the aggregate level (panel [1] in figure 3) has to do with the lognormal distribution of life time income. A relatively large share of the low income types is clustered at very low incomes and therefore experiences large utility gains from the targeted transfers that push the LL program ahead even after averaging over all income types.

The same cannot be said for LS programs as here the adverse savings distortions play out stronger against the late redistribution program. A small LS program would still be better for low income types (panel [2] in figure 3) but after aggregating the utility levels over all income types this "dominance" only holds for extremely small targeted LS programs (panel [1] in figure 3).

In figure 5 we illustrate that Gini coefficients are decreasing with the size of the redistribution programs. Conditional on using labor taxes to finance redistribution, we find that early redistribution (EL) generates smaller Gini coefficients than late redistribution. Comparing these results with the welfare results in figure 4 where early redistribution financed by a labor tax (EL) generates the largest welfare increases, we find that early redistribution can be used to enhance utility and equality simultaneously.

5.2 Changing the Targeting Rate of Redistribution

In our next set of experiments we investigate how an increased targeting rate for late redistribution programs (via parameter b_L) can ensure that the late redistribution program stays small and therefore dominates over the early redistribution regimes. We again compare late redistribution programs and early redistribution programs. In these sets of experiments we fix parameter a_L at a very small level $a_L = 0.05$ (~ \$197,159). We need to fix this lump-sum component of the late redistribution program to be small because otherwise the required tax rates to finance the program would be very large and late redistribution programs would always be dominated by early redistribution programs. We do not re-calibrate any of the other parameters.

Figures 6 and 7 report the results of these experiments. We first look at how aggregate output changes as we increase the targeting rate of late redistribution programs b_L. As we change b_L we have to think which other government parameter do we want to adjust to clear the government budget constraint. Since we already fixed a_L at a level of 0.05 the only other parameters left are the labor tax rate τ^L for the regimes financed by labor taxes (EL, LL) and the capital tax rate τ^K for regimes finance by capital taxes (ES, LS). We see that as we increase b_L and the late redistribution programs become more targeted, the necessary taxes to finance them can be reduced (compare panel [7] in figure 6). This endogenous adjustment happens automatically for late redistribution programs due to budget balancing. In order to compare the late redistribution regimes to the early redistribution regimes we will set the tax rates of the early redistribution regime equal to the endogenously adjusting tax rate of the late redistribution regime. So the labor tax rate for EL is set equal to the endogenously adjusted labor tax rate of regime LL, whereas the capital tax rate of regime ES is set equal to the endogenously adjusted capital tax rate of regime LS (see also panel [8] in figure 6).

From panel [1] in figure 6 we see that as the late redistribution regimes become more targeted and the respective volume of redistribution decreases, output increases. The opposite is true for the early redistribution programs. If the tax rates of early redistribution programs mirror the decrease of their late redistribution counterparts, the "engine of growth" of early transfers begins to stall and output declines. In terms of output we get the same result as before. Late redistribution programs will always be dominated by early redistribution programs, no matter how targeted late redistribution programs become.

In terms of welfare the picture changes again. Figure 7 shows that as the targeting of late redistribution increases and the programs become smaller, aggregate welfare in these regimes increases, due to smaller distortions and more aggressive redistribution. On the other hand, as we adjust the size of early redistribution programs at the same rate as the late redistribution programs we see that welfare decreases for regime EL and ES. There is a threshold targeting parameter around $b_L = 0.08$ after which programs are small enough so that the late redistribution regimes start dominating the early redistribution regimes in terms of aggregate welfare. The effects are more pronounced for low income individuals as can be seen in panel [2] of figure 7. For low income groups the targeting threshold after which the dominance switches from early to late redistribution programs is much smaller at $b_L = 0.05$. Figure 8 reports welfare per income quartiles. The Gini coefficient in panel [6] of figure 8 indicates that wealth inequality increases despite the increased rate of targeting. This is a direct consequence of the shrunken transfer programs due to stronger targeting.

We once more conclude that more targeted programs, that can therefore be kept small in size are able to dominate early redistribution programs in terms of aggregate welfare but not in terms of output.

5.3 Sensitivity Analysis and Limitations

Sensitivity Analysis. We conduct the same set of experiments for a model with inelastic labor supply. Our results are robust to this modification. However, in the case with inelastic labor supply the late redistribution programs have an even harder time to dominate the early redistribution programs. The reason is that early redistribution programs suffer more in general from distortions in the labor market and once we turn those off there is only a very small region in the policy parameter space left where late redistribution programs can dominate early redistribution programs.

Our results are also robust to changes in the discount factor. We tried discount factors in the range of $\beta = \{0.94, ..., 0.988\}$ and found that small sized late redistribution programs can dominate early redistribution programs in terms of welfare.[12] With larger discount factors even somewhat larger late redistribution programs can dominate the early ones.

We also ran the experiments for a range of the income distribution parameters $\mu = \{441,000,\ \mathbf{683,000},\ 806,000\}$ and $\sigma_I = \{0.34,\ \mathbf{0.37},\ 0.40\}$ where the bold numbers indicate benchmark values. All results are robust with respect to changes of μ and σ_I. The threshold for the size of labor/capital tax after which the early redistribution programs dominate the late redistribution programs change slightly for the lowest income group but still hold qualitatively.

Additional sensitivity analysis was conducted for the preference parameters $\sigma = \{2.3,\ \mathbf{2.5},\ 2.7\}$ and $\eta = \{0.34,\ \mathbf{0.37},\ 0.40\}$ where the bold numbers again represent benchmark values. We find that our results hold for alternative values of σ and η. The critical tax levels at which the

[12] Annual discount factors β translate into per period discount factors as $\beta_{period} = \beta^{30}$. The range of per period discount factors is accordingly $\beta_{period} = \{0.1563, ..., 0.6961\}$.

"domination" of late redistribution programs is broken are very similar in these experiments. The only values that do change significantly are the welfare levels. So qualitatively the results are robust across reasonable changes in preference parameters. Quantitatively the economies operate at slightly different levels. When it comes to determining the critical size of redistribution though, the results are remarkably similar across all tested values of σ and η. All results of these sensitivity experiments are available in a technical appendix upon request from the authors.

Limitations. Agents face no uncertainty and are prohibited from giving private intergenerational transfers like bequests. In addition agents do face borrowing limits. However, it should be pointed out that in the benchmark calibration transfers to young agents are large enough so that no agent will ever hit the borrowing limit, i.e. $s > 0$ for all agents. This result is driven by the 2-period structure of the model, where one period spans roughly 30 years and it is therefore unlikely that an agent will need to borrow over such a long time. In a more comprehensive multi period model a la Auerbach and Kotlikoff (1987), binding borrowing constraints would be more likely and a model imposing binding borrowing constraints would strengthen the case for early redistribution programs compared to models that freely allow borrowing. A similar case could be made for models that allow for idiosyncratic income or spending shocks (i.e. unemployment shocks, health spending shocks, etc.) where a model with borrowing constraints in place would value early redistribution programs higher than a model without borrowing constraints.

6 Optimal Redistribution and Tax Policy

We define optimal policy as a set of government policy parameters $a_E, b_E, a_L, b_L, \tau^L, \tau^K$, and λ that maximize total steady state welfare (27) such that the conditions for competitive equilibrium hold. Total welfare is defined as the equally weighted sum of the lifetime utilities of all individuals who are newly born into the steady state. More formally, the government maximizes the utilitarian welfare function

$$\begin{aligned}
&\max_{\{a,b,\tau^L,\tau^K,\lambda\}} \int_{\underline{\theta}}^{\bar{\theta}} \left[u\left(c^y\left(\theta\right), l\left(\theta\right)\right) + \beta u\left(c^o\left(\theta\right)\right)\right] dF\left(\theta\right) \\
&s.t. \\
&(5), (7), (13)-(16), (18), (19), (20), (21), (22), \text{ and } (23).
\end{aligned}$$

We abstract completely from transitional issues and report our results in table 8. We are therefore not able to claim that the optimal policies outlined below are Pareto improving.

Our first finding, shown in table 8 as model $M1$, is that using the most general version of our model with agent heterogeneity, elastic labor supply, and non-separable utilities the optimal government transfer and tax policy for the U.S. economy is: $a_E = 0.86$ (˜\$672,000), $b_E = 0.19$, $\tau^L = 53\%$, $\tau^K = 100\%$, and $\lambda = 1$. Since all transfers go to the young generation the redistribution parameters of the late transfer regime a_L and b_L are undefined.

Compared to the original calibration of the U.S. economy our model implies that transfers to the young should be increased from currently 38 percent of all tax revenue to 100 percent. In addition, labor taxes should increase from 27 percent to 53 percent and capital taxes should increase from 17 percent to 100 percent. In addition, the targeting rate decreases from $b_E = 0.30$ to $b_E = 0.19$, so that means testing becomes less aggressive. This policy increases the steady state capital stock by a factor of three, reduces labor supply from 36.7 percent to 29.0 percent and increases output by 50 percent. We interpret this result as the Jones and Manuelli (1992) finding that transfers to the young generation can increase output significantly. The increase in welfare is a direct result of the higher income and the higher consumption of leisure. Note also that capital taxes are large.

As in Auerbach and Kotlikoff (1987) and Jones and Manuelli (1992) the optimal tax policy described above is not welfare enhancing in a Paretian sense as the utility of the initial old generation as well as the utility of the high income earners will be lower with increasing tax rates. Also, our concentration on steady states misses the costs imposed on transitional generations who have to build up the capital stock which comes at the expense of their consumption.[13]

Positive capital tax rates are not unusual in overlapping generations economies as has been shown in the literature (e.g. Ordover and Phelps (1975), Atkinson and Sandmo (1980), Hubbard and Judd (1986), Auerbach and Kotlikoff (1987), Alvarez et al. (1992), Aiyagari (1995), Imrohoroglu (1998), Garriga (2000), Erosa and Gervais (2002), Conesa, Kitao and Krueger (2009), and others). Some of the model features that ensure a positive capital tax rate are borrowing constraints, increasing efficiency profiles, and agent income heterogeneity within age cohorts. We can show that none of these features are essential to achieve a positive capital tax rate as the optimal government financing rule.

In order to give more intuition we use a simplified version of our model, where agents supply labor inelastically, they are homogenous within their age cohort, and capital and labor taxes are levied to finance an exogenously given government consumption equal to $\overline{Tax} > 0$. We can derive the steady state capital stock as

$$K = \frac{\left(1-\tau^L\right)(1-\alpha)AK^{\alpha}}{1+\beta^{-\frac{1}{\sigma}}\left(1+\left(1-\tau^K\right)\left(\alpha AK^{\alpha-1}-\delta\right)\right)^{1-\frac{1}{\sigma}}},$$

and after substituting the government budget constraint into the above equation we have

$$K = \frac{(1-\alpha)AK^{\alpha}-\overline{Tax}+\tau^K\left(\alpha^2AK^{\alpha}-\delta K\right)}{1+\beta^{-\frac{1}{\sigma}}\left(1+\left(1-\tau^K\right)\left(\alpha AK^{\alpha-1}-\delta\right)\right)^{1-\frac{1}{\sigma}}}.$$

We can now show that if $\overline{Tax}$ is small and σ is sufficiently large (e.g. $\sigma > 1-\varepsilon$, with ε being a small number) then an increase in τ^K with a corresponding decrease in τ^L leads to a higher steady state capital stock. The mechanism at work follows the intuition given in Jones and Manuelli (1992) and can be described as follows.

Even though we abstract from economic growth, we find that young agents with higher incomes are able to buy more capital stock from the old generation. Whenever τ^K increases and government consumption is held constant, then τ^L decreases. This leads to higher income of the young agent (income effect). At the same time the rate of return on capital decreases (substitution effect). It turns out that under certain parameter restrictions (small government program, sufficient curvature on preferences), an increase in the capital tax increases the capital stock as the income effect dominates the substitution effect. This result is robust in models with representative agents, additively separable preferences in consumption and leisure, and variations in the time discount factor.[14]

[13] Apart from computational difficulties that transitions would imply, we would like to compare our results to the existing literature on optimal taxation which is concentrated on steady state analysis.

[14] Some more intuition can be gained by noting that in our model the young generation is taxed via a labor tax and the old generation is taxed via a capital tax only. Since it has been shown in the literature that it is optimal to tax income at different ages at different tax rates (e.g. Garriga (2000), and Erosa and Gervais (2002)) we now see that the only way to achieve more inter generational equality is by taxing the old generation as well. However, in our model we assume the old generation is not working anymore so that the only way to tax the old is via the capital tax. This provides a further justification for a positive capital tax rate. Conesa, Kitao

Finally we conduct sensitivity analysis on the optimal policies given various alternative model specifications summarized in table 8. We find that the optimal policy results of transferring to the young generation only and taxing interest income at 100 percent are very robust across alternative specifications.

7 Conclusion

We study a simple two-period OLG model with endowment heterogeneity under five policy regimes, (*i*) no redistribution, (*ii*) early redistribution to the young generation financed by taxes on wage income of the current young generation, (*iii*) early redistribution to the young generation financed by taxes on interest income of the current old generation,(*iv*) late redistribution to the old generation financed by taxes on wage income of the current young generation, and (*v*) late redistribution to the old generation financed by taxes on interest income of the current old generation.

The main finding of this paper is that late redistribution can dominate early redistribution only when late redistribution is small. If late redistribution is large, we find that early redistribution dominates since the savings distortion of late redistribution becomes too large. We find that high income households fare best under the benchmark model, whereas the welfare of very low income households can be improved under various redistribution regimes. Low income households seem to do better under late redistribution, if tax rates are not too high. Output is highest under the early redistribution regime with inter generational financing (savings tax on the old). Optimal tax policy points towards a non-zero tax on capital and an emphasis on early redistribution.

Like all theoretical analyses our paper has relied on a set of simplifying assumptions. First, we do not take into consideration private transfers from the old to the young. These kinds of transfers can take the shape of financing education of the young, at the end of life transferring real property to the young or outright cash gifts. Such transfers to the young would increase wealth of the young and hence increase savings rates. In turn the presence of these kinds of transfers would tend to decrease the welfare arguments for early distribution. The quantitative impact of these types of transfers on the results would of course depend on the size of the transfers and where along the wealth/ability distribution these transfers occur.

In our model there is one type of "uncertainty", ability shocks that are realized in the first stage of life after the government's transfers to the young but before government transfers to the old. This random assignment of innate ability or earnings capacity is intended to capture a vast variety of life's circumstances and uncertainties. Of course these uncertainties can be broken down into uncertain earnings, health (expenditure) shocks, uncertainty over the length of life and others. To the extent that the realizations of the random variables occur at different parts of the life cycle they would have a tendency to influence the results. Health and mortality shocks tend to occur towards the end of life. Large uncertainty through health or mortality shocks at the end of life tilt the welfare balance in favor or late redistributive transfers as such transfers are able to provide better insurance. Of course, when the government provides insurance against late in life health shocks, private self-insurance through savings will adjust as well. We leave these important questions for future research.

and Krueger (2009) show that if the old generation is working and an age specific labor tax rate is present then the optimal capital tax rate can be zero. In this case the progressivity of the labor tax rate would achieve the optimally feasible inter generational equality and capital tax would not be needed to reach the optimum.

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8 Appendix: Tables and Figures

Table 1: Government Transfers divided into transfers to the young and the old.

Selected Government Spending in 2004: in Billions of $

(1,955 represents $1,955,000,000,000)

Selected spending, total		**$1,955**
	Early Transfers	**$746**
Federal:		
	Unemployment assistance	$43
	SCHIP	$5
	Medicaid (69%)	$122
	Health other (69%)	$23
	Housing assistance (66.6%)	$21
	Foodstamps (91%)	$32
	Child nutrition	$11
	Earned income tax credit	$33
	Daycare and foster care	$11
	Higher education	$25
	Research and general education aid	$3
	Training and employment	$8
State:		
	Higher education	$154
	Other education	$37
	Public Welfare/Hospitals/Health (69%)	$176
	Unemployment compensation	$43
	Late Transfers	**$1,210**
Federal:		
	Social Security+Medicare	$792
	Federal employees retirement	$117
	Medicaid (31%)	$55
	Health other (31%)	$10
	Uniformed services retiree health care fund	$5
	Housing assistance (33.4%)	$10
	Foodstamps (9%)	$3
State:		
	Public welfare/hospitals/health (31%)	$79
	Employee retirement	$138
Residual Government spending		$2,172
Total government expenditures		$4,127
Size of selected spending as fraction of GDP		17%
Size of total government spending as fraction of GDP		35%
Early transfers in percent of total selected spending (λ)		38%
Late transfers in percent of total selected spending ($1-\lambda$)		62%

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html
Source for Federal:http://www.irs.gov/taxstats/article/0,,id=168610,00.html

Table 2: Parameters.

Parameters		Source/Moment to match
Preferences		
Inv. of elast. of substitution	$\sigma = 2.5$	to match K/Y and r
Time preference	$\beta = 0.953$	to match K/Y and r
Consumption preference	$\eta = 0.37$	to match lifetime labor supply $\%L$
Leisure preference	$1 - \eta = 0.63$	to match lifetime labor supply $\%L$
Technology		
Total factor productivity	$A = 2.98$	to normalize wages $w = 1$
Capital share in production	$\alpha = 0.36$	$0.30 - 0.36$ are standard values (e.g. Kydland and Prescott (1982))
Annual discount rate	$\delta = 8\%$	$5.9\% - 12\%$ in Nadiri and Prucha (1996)
Ability Distribution		
Log of lifetime income	$\mu = 13.43$ (˜$683,000)	Fullerton and Rogers (1993) and own calculations
Stand. dev. of lifetime income	$\sigma_I = 0.415$	
Lowest lifetime income	$\underline{\theta} = 0.004$ (˜$10,000)	
Highest lifetime income	$\bar{\theta} = 121$ (˜$5.000,000)	
Policy Parameters		
Wage income tax rate	$\tau^L = 27.0\%$	to match VR and $\%\tau^L$
Capital tax rate	$\tau^K = 17\%$	to match VR and $\%\tau^L$
Targeting parameter	$b_E = 0.30$	
Max late transfer	$a_E = 0.19$ (˜$346,000)	
Targeting parameter	$b_L = 0.20$	
Max late transfer	$a_L = 0.22$ (˜$361,000)	
Share of early redistribution	$\lambda = 38\%$	U.S. Stat.Abstr. 2004 & own calculations

Table 3: Matched Moments

Matched Moments		
Annual interest rate	$r = 3.79\%$	4%in NIPA accounts
Capital output ratio	$K/Y = 3.18$	$2.7 - 3$ are standard from NIPA accounts
Average Lifetime Labor Supply	$\%L = 0.36$	0.374 in Gomes, Kotlikoff and Viceira (2007)
Percent of labor tax revenue	$\%\tau^L = 72\%$	75% in U.S. Stat.Abstr. 2004 & own calculations
Percent of capital tax revenue	$\%\tau^K = 28\%$	25% in U.S. Stat.Abstr. 2004 & own calculations
Size of government	$VR = 21.7\%$	21.4% in U.S. Stat.Abstr. 2004 & own calculations

Table 5: Tax Revenue 2004

Tax Revenue Fiscal Year 2004: in Billions of $

(3,029 represents $ 3,029,000,000,000)

United States, total:			**$3,029**
Federal Tax Income			$2,019
	Individual income tax	$990	
	withheld by employers	*$747*	
	Employment tax	$717	
	Old-age and disability insurance	*$706*	
	Unemployment insurance	*$7*	
	Railroad retirement	*$4*	
	Corporation income tax	$231	
	Estate and gift tax	$26	
	Excise tax	$55	
State and Local Taxes			$1,010
	Property	$318	
	Individual income tax	$215	
	Corporation income	$34	
	Sales and gross receipt	$361	
	Motor vehicle licenses	$21	
	Death and gift	$6	
	other	$56	

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html
Source for Federal:http://www.irs.gov/taxstats/article/0,,id=168610,00.html

Table 6: Shares of labor tax revenue and capital tax revenue as percentage of total tax revenue excluding consumption, sales and excise taxes.

Tax Split into Labor and Capital Tax: in Billions of $

(2,415 represents $ 2,415,000,000,000)

Total Tax Revenue, excl. consumption taxes			**$2,415**
	Labor Tax		
Federal:			
	Individual income tax (employer)	$747	
	Individual income tax (employee)	$122	
	Employment tax	$717	
State:			
	Individual income tax (state)	$215	
Wage income tax revenue:			**$1,801**
(in %)			**75%**
	Capital Tax		
Federal:			
	Corporation income Tax	$231	
	Estate and gift tax	$26	
State:			
	Property tax	$318	
	Corporation income tax	$34	
	Death and gift tax	$6	
Capital income tax revenue:			**$615**
(in %)			**25%**

Source for State and Local:

http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html

Source for Federal:http://www.irs.gov/taxstats/article/0,,id=168610,00.html

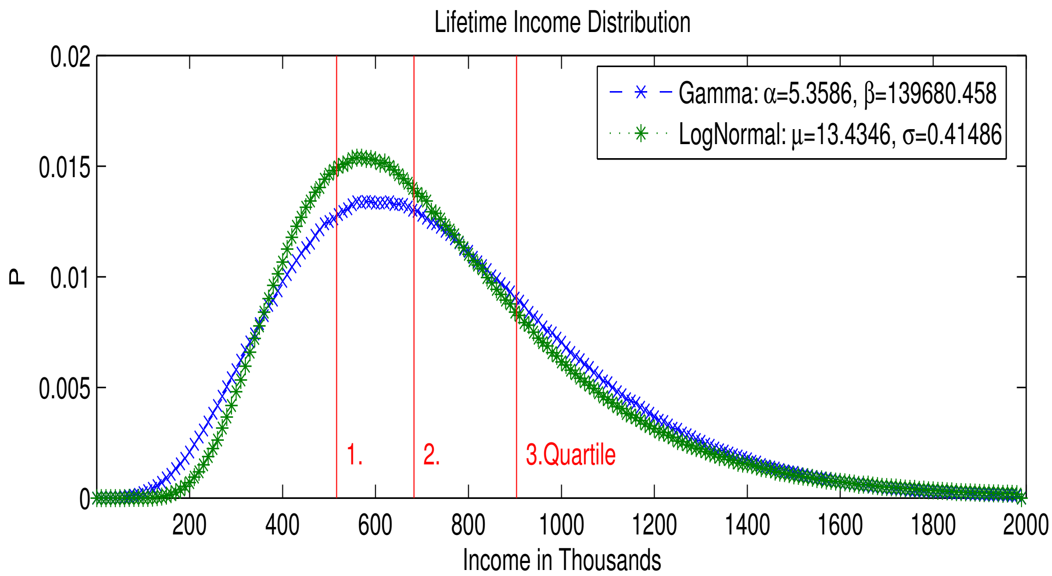
Table 7:
Government Expenditures Fiscal Year 2004: in Billions of $
(2,293 represents $ 2,293,000,000,000).

United States, total:		**$4,127**
Federal Spending		$2,293
Social Security		$496
Federal employees retirement		$117
Unemployment assistance		$43
Medical Care		$515
Medicare	*$296*	
SCHIP	*$5*	
Medicaid	*$176*	
Indian Health	*$3*	
Hospital and medical care for veterans	*$22*	
Health resources and services	*$6*	
Substance abuse and mental health services	*$3*	
Health care tax credit	*$0*	
Uniformed Services retiree health care fund	*$5*	
Housing assistance		$31
Food and nutrition assistance		$46
Foodstamps	*$35*	
Child nutrition and special milk programs	*$11*	
Public assistance		$112
Earned income tax credit	*$33*	
Supplemental security income	*$31*	
Daycare and foster care	*$11*	
other	*$36*	
Other payments to individuals		$12
Education, training, employment and social services		$88
Elementary and secondary	*$34*	
Higher education	*$25*	
Research and general education aid	*$3*	
Training and employment	*$8*	
Social services	*$16*	
others (less transfers to State/Local)		$408
State and Local Spending (net of federal funds)		$1,834
Education (net of federal funds)		$584
Elementary and secondary	*$393*	
Higher education	*$154*	
Other education	*$37*	
Public Welfare (net of federal funds)		$118
Health and Hospitals (net of federal funds)		$137
Utility & liquore store Expenditure		$160
Insurance trust expenditure		$197
Employee Retirement	*$138*	
Unemployment compensation	*$43*	
Other infrastructure		$638

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html
Source for Federal:http://www.irs.gov/taxstats/article/0,,id=168610,00.html

			Policy Parameters						
			a_E	b_E	a_L	b_L	τ^L	τ^K	λ
U.S.A.			0.19	0.30	0.22	0.20	27%	17%	0.38
	Optimal Policy								
Non-separable utility	Income hetero-geneity	Elastic Labor Supply							
$M\ 1:$	yes	yes	0.87	0.27	–	–	60%	100%	1
$M\ 1b : \beta = 0.985$	yes	yes	0.95	0.25	–	–	59%	100%	1
$M\ 2:$	yes	no	1.12	0.26	–	–	100%	100%	1
$M\ 3:$	no	yes	8.77	0	–	–	0%	100%	1
$M\ 4:$	no	no	–	–	–	–	–	100%	1
Separable utility									
$M\ 5:$	yes	yes	.99	0.6	–	–	62%	51%	1
$M\ 6:$	yes	no	.76	0.26	–	–	78%	100%	1
$M\ 7:$	no	yes	8.97	0	–	–	0%	100%	1
$M\ 8:$	no	no	–	–	–	–	–	42%	1

Table 8: Optimal tax policy. If cells contain $-$ then we have indeterminacy. When $\lambda = 1$ then we only have early redistribution in the system and the parameters for late redistribution become irrelevant. In model M4 and M8 the optimal outcome is independent of the amount of a lump-sum transfer to the young financed by a lump sum tax on the young (i.e. in the model with inelastic labor supply, labor taxes become lump sum taxes when agents are not heterogenous). We restrict the search of optimal tax rates τ_L and τ_K to within zero and 100 percent.



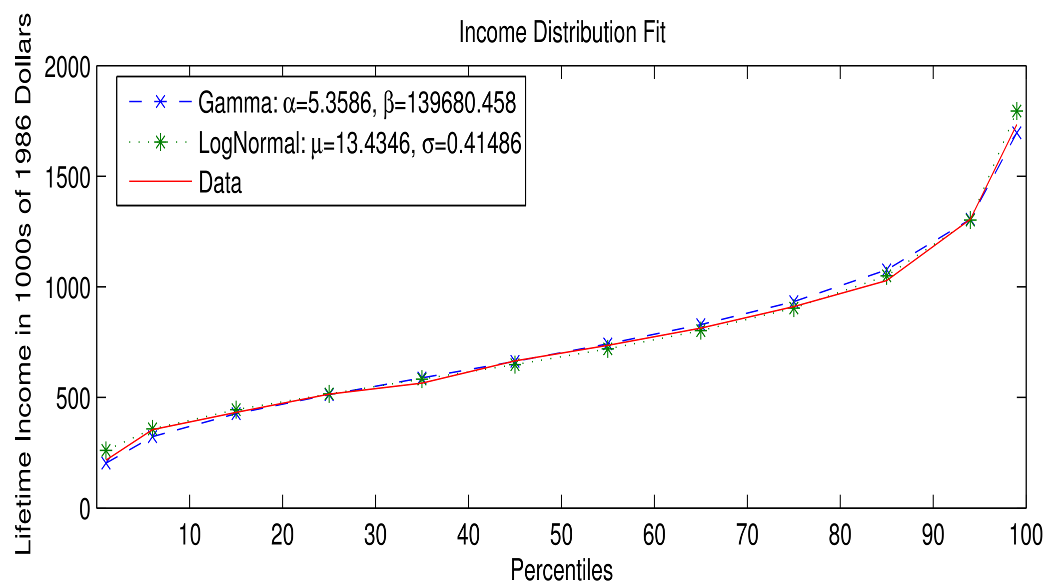


Figure 1: Fullerton and Rogers (1993).

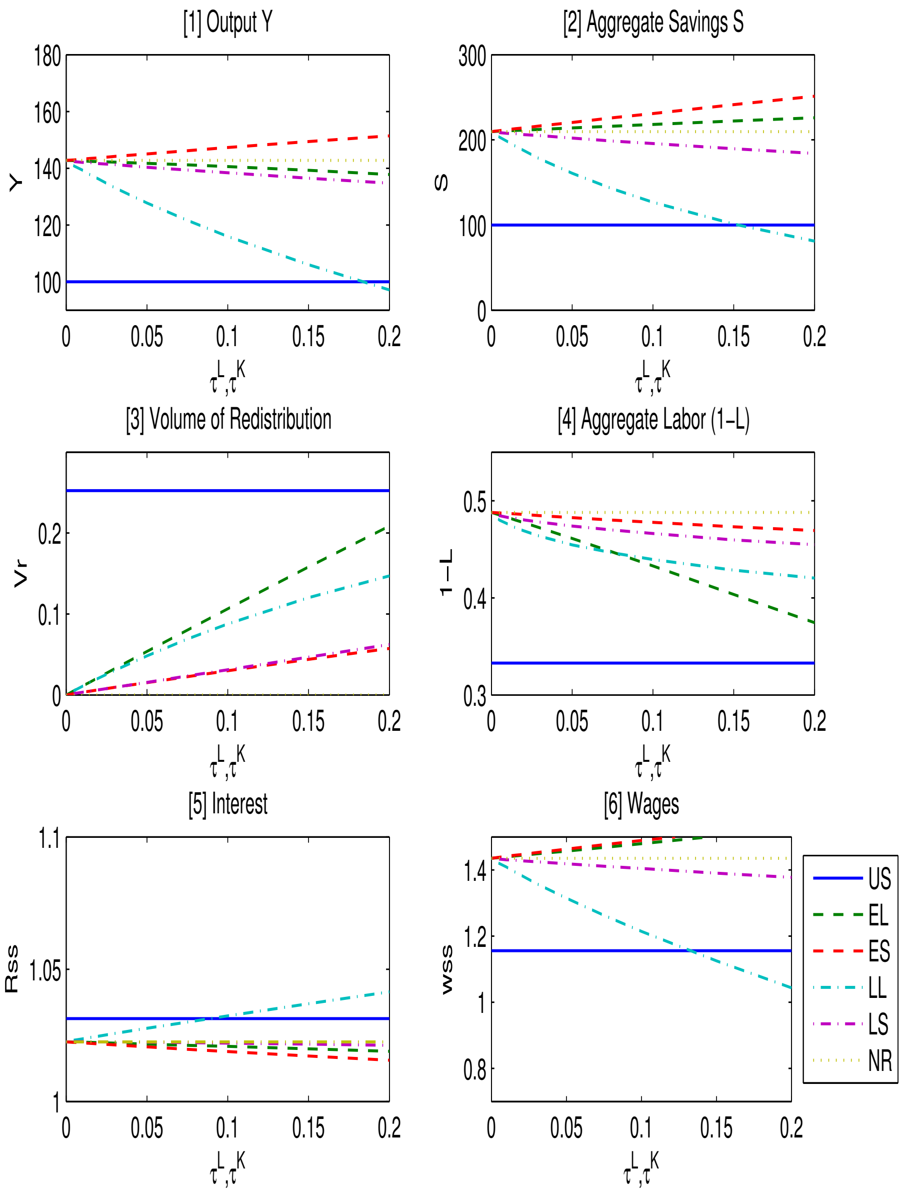


Figure 2: Steady State Results for Size Adjusted Government Redistribution Programs.

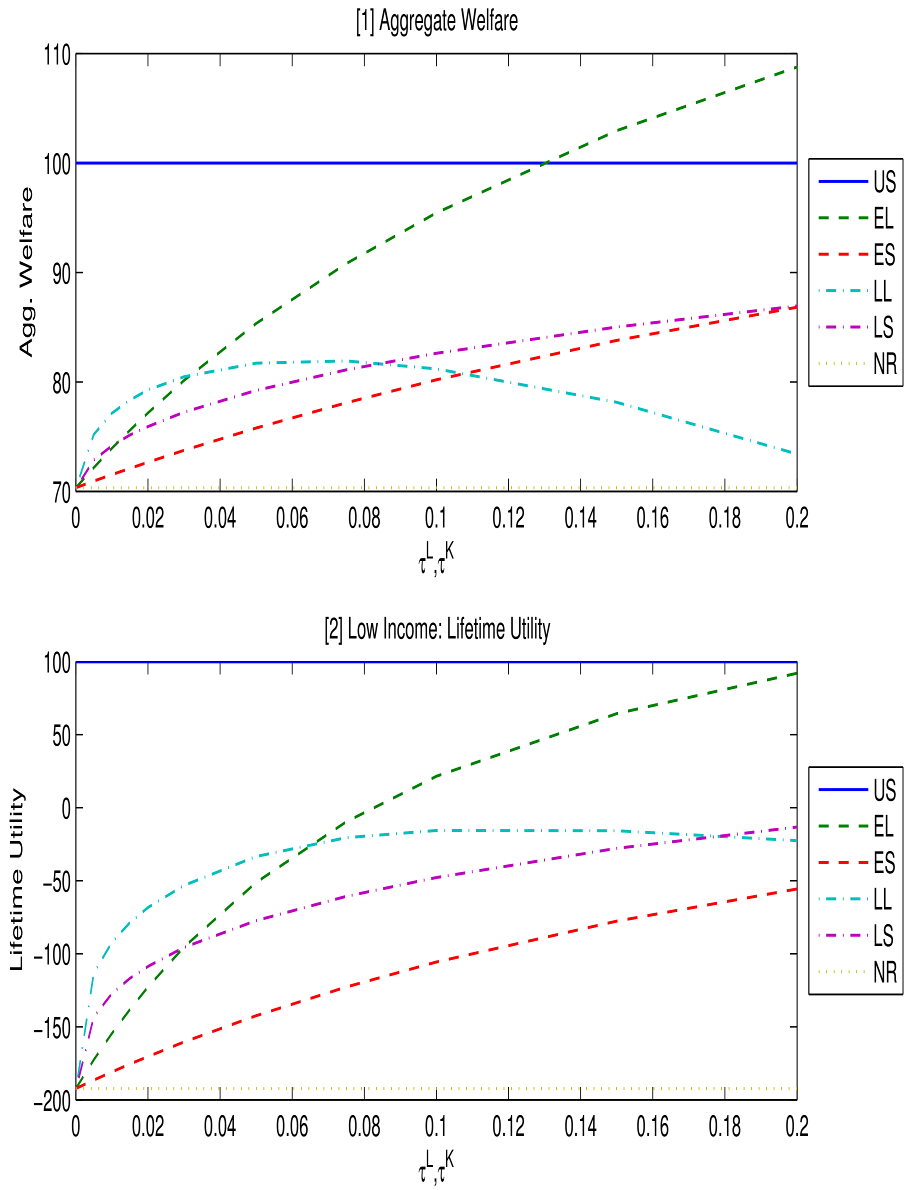


Figure 3: Aggregate Welfare and Welfare of the Poorest Household for Size Adjusted Government Redistribution Programs.

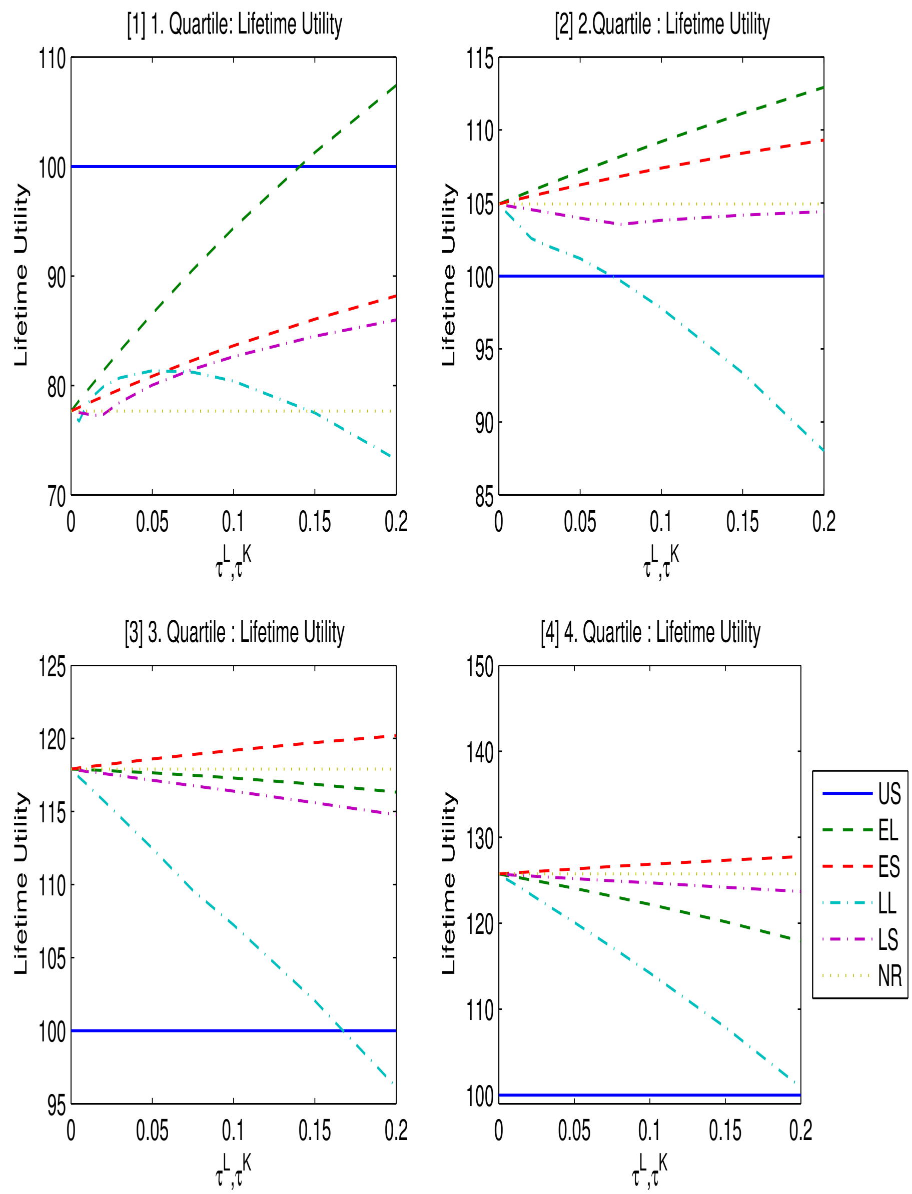


Figure 4: Welfare per Income Quartile for Size Adjusted Government Redistribution Programs.

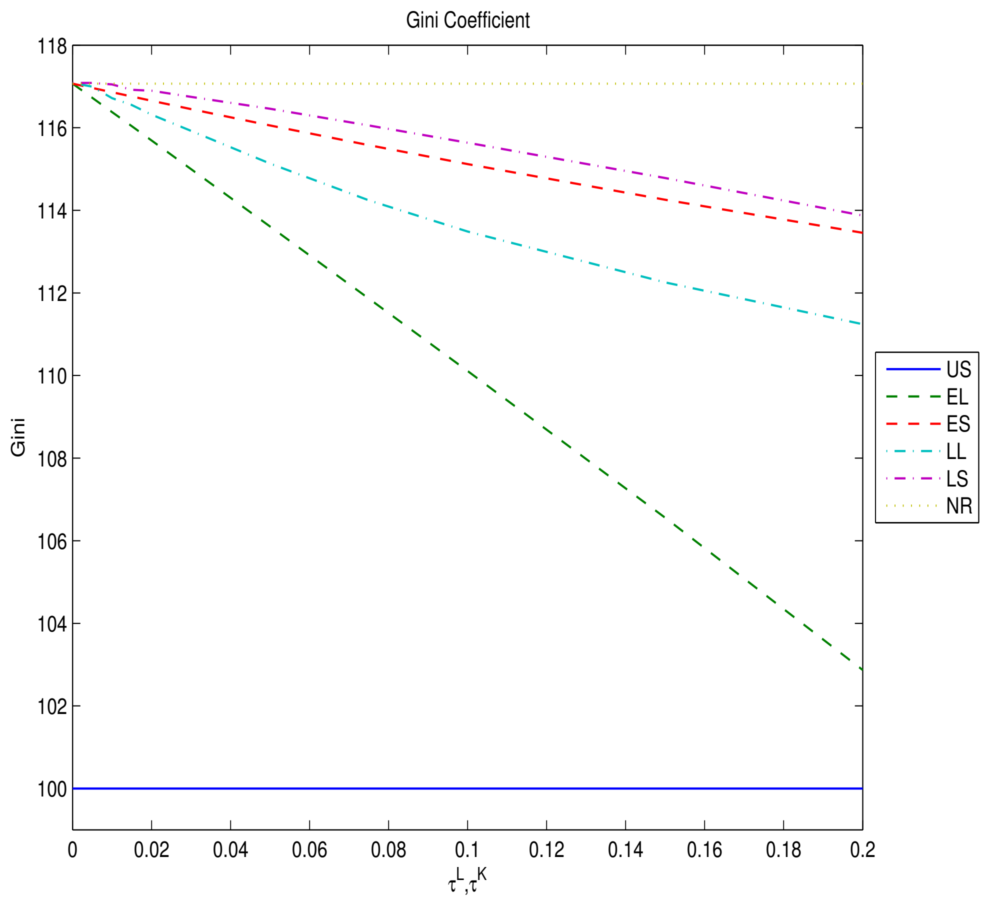


Figure 5: Gini Coefficient

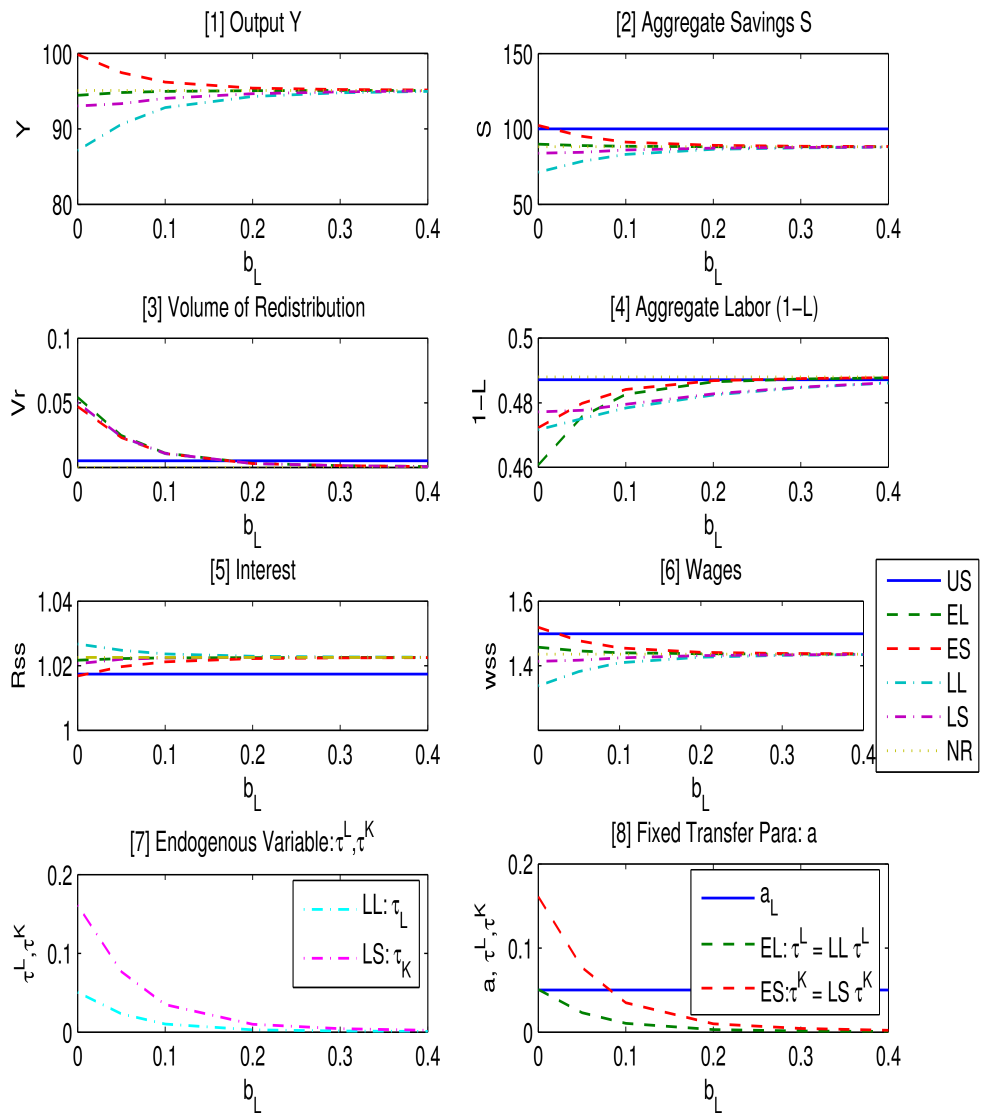


Figure 6: Steady State Results for Differently Targeted Late Transfer Programs and Size Adjusted Early Transfer Programs.

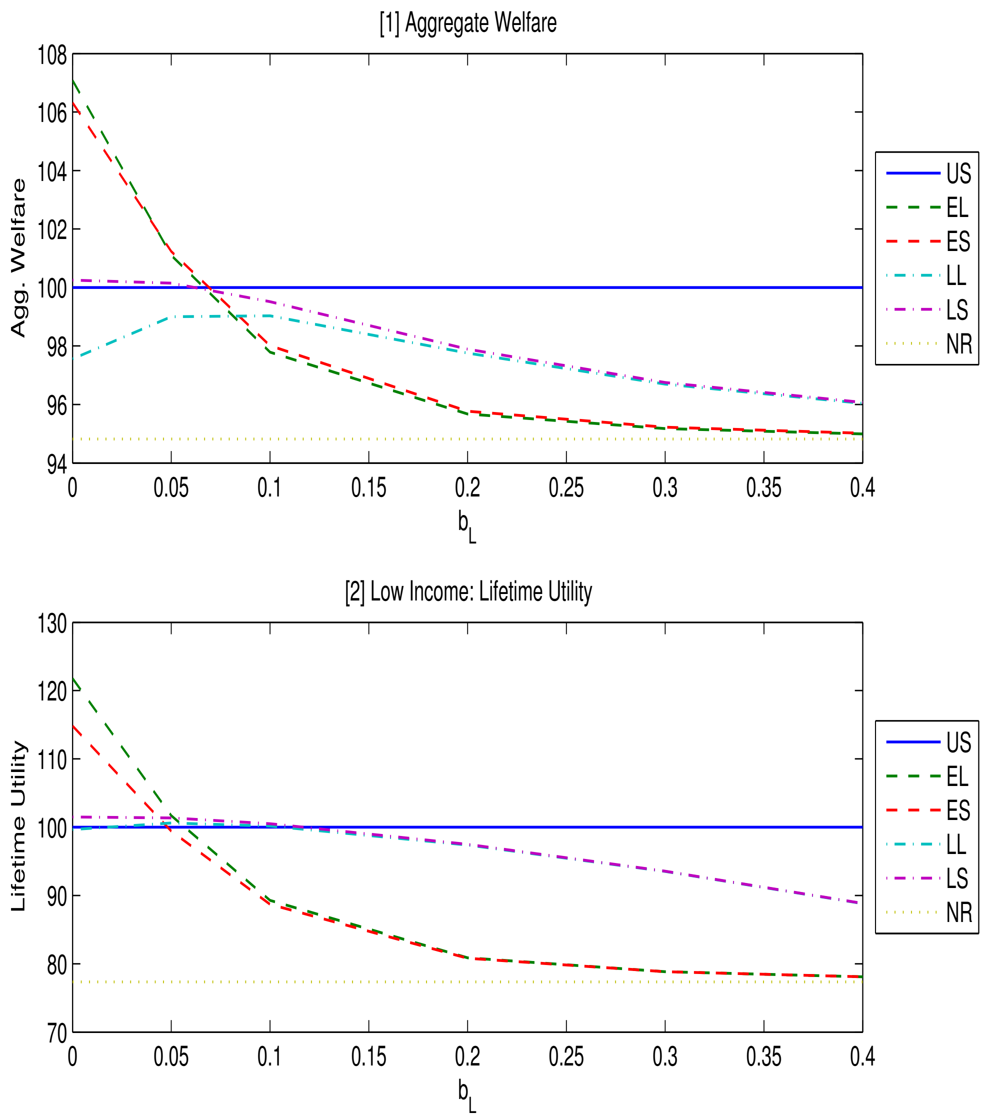


Figure 7: Aggregate Welfare and Welfare of the Poorest Household for Differently Targeted Late Transfer Programs and Size Adjusted Early Transfer Programs.

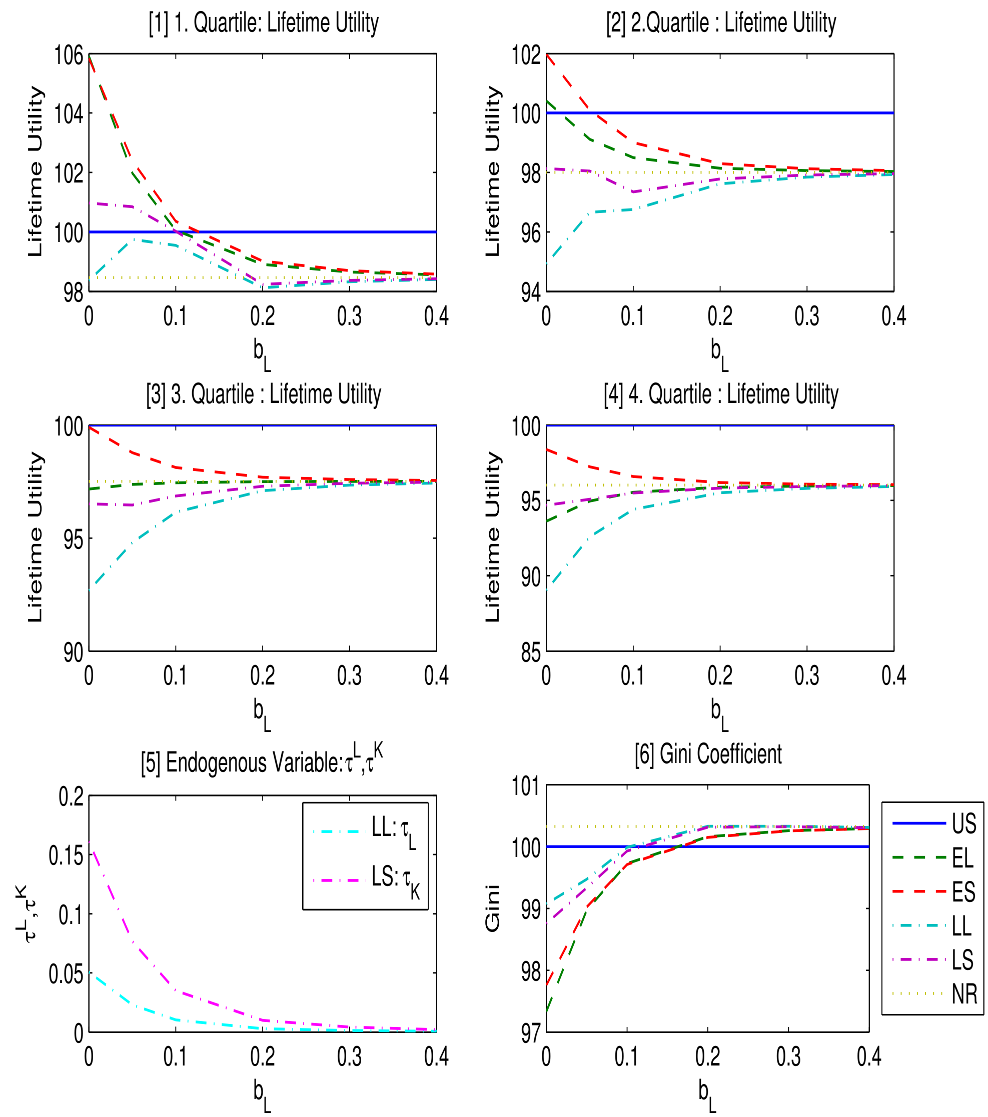


Figure 8: Welfare per Income Quartile for Differently Targeted Late Transfer Programs and Size Adjusted Early Transfer Programs.