

## Confidence Intervals and Hypothesis Testing

### Multiple Choice

Identify the choice that best completes the statement or answers the question.

- \_\_\_\_\_ 1. The librarian at the Library of Congress has asked her assistant for an interval estimate of the mean number of books checked out each day. The assistant took a sample and found the mean to be 880 books. She provides the librarian with an interval estimate of between 790 and 970 books checked out per day. An efficient, unbiased point estimate of the number of books checked out each day at the Library of Congress is:
- 790
  - 880
  - 90
  - None of these choices.
- \_\_\_\_\_ 2. After constructing a confidence interval estimate for a population mean, you believe that the interval is useless because it is too wide. In order to correct this problem, you need to:
- increase the population standard deviation.
  - increase the sample size.
  - increase the level of confidence.
  - increase the sample mean.
- \_\_\_\_\_ 3. Suppose an interval estimate for the population mean was 62.84 to 69.46. The population standard deviation was assumed to be 6.50, and a sample of 100 observations was used. The mean of the sample was:
- 56.34
  - 62.96
  - 6.62
  - 66.15
- \_\_\_\_\_ 4. The sample size needed to estimate a population mean within 2 units with a 95% confidence when the population standard deviation equals 8 is
- 61
  - 62
  - 8
  - None of these choices.
- \_\_\_\_\_ 5. A Type I error is committed if we make:
- a correct decision when the null hypothesis is false.
  - a correct decision when the null hypothesis is true.
  - an incorrect decision when the null hypothesis is false.
  - an incorrect decision when the null hypothesis is true.
- \_\_\_\_\_ 6. The hypothesis of most interest to the researcher is:
- the alternative hypothesis.
  - the null hypothesis.
  - both hypotheses are of equal interest.
  - Neither hypothesis is of interest.

- \_\_\_\_\_ 7. A Type II error is defined as:
- rejecting a true null hypothesis.
  - rejecting a false null hypothesis.
  - not rejecting a true null hypothesis.
  - not rejecting a false null hypothesis.
- \_\_\_\_\_ 8. Which of the following statements is not true?
- The probability of making a Type II error increases as the probability of making a Type I error decreases.
  - The probability of making a Type II error and the level of significance are the same.
  - The power of the test decreases as the level of significance decreases.
  - All of these choices are true.
- \_\_\_\_\_ 9. Researchers claim that 60 tissues is the average number of tissues a person uses during the course of a cold. The company who makes Kleenex brand tissues thinks that fewer of their tissues are needed. What are their null and alternative hypotheses?
- $H_0: \mu = 60$  vs.  $H_1: \mu > 60$
  - $H_0: \mu = 60$  vs.  $H_1: \mu < 60$
  - $H_0: \bar{X} = 60$  vs.  $H_1: \bar{X} < 60$
  - $H_0: \mu < 60$  vs.  $H_1: \mu = 60$
- \_\_\_\_\_ 10. In testing the hypotheses  $H_0: \mu = 50$  vs.  $H_1: \mu \neq 50$ , the following information is known:  $n = 64$ ,  $\bar{x} = 53.5$ , and  $\sigma = 10$ . The standardized test statistic  $z$  equals:
- 1.96
  - 2.80
  - 2.80
  - 1.96
- \_\_\_\_\_ 11. If a hypothesis is not rejected at the 0.10 level of significance, it:
- must be rejected at the 0.05 level.
  - may be rejected at the 0.05 level.
  - will not be rejected at the 0.05 level.
  - must be rejected at the 0.025 level.
- \_\_\_\_\_ 12. In testing the hypotheses  $H_0: \mu = 75$  vs.  $H_1: \mu < 75$ , if the value of the test statistic  $z$  equals  $-2.42$ , then the  $p$ -value is:
- 0.5078
  - 2.4200
  - 0.9922
  - 0.0078
- \_\_\_\_\_ 13. For a two-tail test, the null hypothesis will be rejected at the 0.05 level of significance if the value of the standardized test statistic  $z$  is:
- smaller than 1.96 or greater than  $-1.96$
  - greater than  $-1.96$  or smaller than 1.96
  - smaller than  $-1.96$  or greater than 1.96
  - greater than 1.645 or less than  $-1.645$

- \_\_\_\_\_ 14. If a hypothesis is rejected at the 0.025 level of significance, it:
- must be rejected at any level.
  - must be rejected at the 0.01 level.
  - must not be rejected at the 0.01 level.
  - may or may not be rejected at the 0.01 level.
- \_\_\_\_\_ 15. Which of the following  $p$ -values will lead us to reject the null hypothesis if the level of significance equals 0.05?
- 0.150
  - 0.100
  - 0.051
  - 0.025
- \_\_\_\_\_ 16. Suppose that we reject a null hypothesis at the 0.05 level of significance. Then for which of the following  $\alpha$ -values do we also reject the null hypothesis?
- 0.06
  - 0.04
  - 0.03
  - 0.02
- \_\_\_\_\_ 17. Suppose that in a certain hypothesis test the null hypothesis is rejected at the .10 level; it is also rejected at the .05 level; however it cannot be rejected at the .01 level. The most accurate statement that can be made about the  $p$ -value for this test is that:
- $p$ -value = 0.01.
  - $p$ -value = 0.10.
  - $0.01 < p$ -value  $< 0.05$ .
  - $0.05 < p$ -value  $< 0.10$ .
- \_\_\_\_\_ 18. If the  $p$  value is less than  $\alpha$  in a two-tail test:
- the null hypothesis should not be rejected.
  - the null hypothesis should be rejected.
  - a one-tail test should be used.
  - No conclusion should be reached.
- \_\_\_\_\_ 19. If an economist wishes to determine whether there is evidence that average family income in a community exceeds \$32,000:
- either a one-tail or two-tail test could be used with equivalent results.
  - a one-tail test should be utilized.
  - a two-tail test should be utilized.
  - None of these choices.
- \_\_\_\_\_ 20. The rejection region for testing  $H_0: \mu = 100$  vs.  $H_1: \mu \neq 100$ , at the 0.05 level of significance is:
- $|z| < 0.95$
  - $|z| > 1.96$
  - $z > 1.65$
  - $z < 2.33$

- \_\_\_\_\_ 21. The owner of a local nightclub has recently surveyed a random sample of  $n = 300$  customers of the club. She would now like to determine whether or not the mean age of her customers is over 35. If so, she plans to alter the entertainment to appeal to an older crowd. If not, no entertainment changes will be made. Suppose she found that the sample mean was 35.5 years and the population standard deviation was 5 years. What is the  $p$ -value associated with the test statistic?
- 0.9582
  - 1.7300
  - 0.0418
  - 0.0836
- \_\_\_\_\_ 22. If the probability of committing a Type I error for a given test is decreased, then for a fixed sample size  $n$ , the probability of committing a Type II error will:
- decrease.
  - increase.
  - stay the same.
  - Not enough information to tell.
- \_\_\_\_\_ 23. The power of a test is denoted by:
- $\alpha$
  - $\beta$
  - $1 - \alpha$
  - $1 - \beta$
- \_\_\_\_\_ 24. For a given sample size  $n$ , if the level of significance  $\alpha$  is decreased, the power of the test will:
- increase.
  - decrease.
  - remain the same.
  - Not enough information to tell.
- \_\_\_\_\_ 25. Researchers determined that 60 Kleenex tissues is the average number of tissues used during a cold. Suppose a random sample of 100 Kleenex users yielded the following data on the number of tissues used during a cold:  $\bar{x} = 52$  and  $s = 22$ . Suppose the alternative we wanted to test was  $H_1: \mu < 60$ . The correct rejection region for  $\alpha = 0.05$  is:
- reject  $H_0$  if  $t > 1.6604$ .
  - reject  $H_0$  if  $t < -1.6604$ .
  - reject  $H_0$  if  $t > 1.9842$  or  $Z < -1.9842$ .
  - reject  $H_0$  if  $t < -1.9842$ .
- \_\_\_\_\_ 26. The degrees of freedom for the test statistic for  $\mu$  when  $\sigma$  is unknown is:
- 1
  - $n$
  - $n - 1$
  - None of these choices.

- \_\_\_\_\_ 27. In selecting the sample size to estimate the population proportion  $p$ , if we have no knowledge of even the approximate values of the sample proportion  $\hat{p}$ , we:
- take another sample and estimate  $\hat{p}$ .
  - take two more samples and find the average of their  $\hat{p}$ .
  - let  $\hat{p} = 0.50$ .
  - let  $\hat{p} = 0.95$ .
- \_\_\_\_\_ 28. After calculating the sample size needed to estimate a population proportion to within 0.04, your statistics professor told you the maximum allowable error must be reduced to just .01. If the original calculation led to a sample size of 800, the sample size will now have to be:
- 800
  - 3200
  - 12,800
  - 6400
- \_\_\_\_\_ 29. A survey claims that 9 out of 10 doctors recommend aspirin for their patients with headaches. To test this claim against the alternative that the actual proportion of doctors who recommend aspirin is less than 0.90, a random sample of 100 doctors' results in 83 who indicate that they recommend aspirin. The value of the test statistic in this problem is approximately equal to:
- 1.67
  - 2.33
  - 1.86
  - 0.14

**True/False**

Indicate whether the statement is true or false.

- \_\_\_\_\_ 30. An unbiased estimator is a sample statistic whose expected value equals the population parameter.
- \_\_\_\_\_ 31. The width of the confidence interval estimate of the population mean  $\mu$  is a function of only two quantities: the population standard deviation  $\sigma$  and the sample size  $n$ .
- \_\_\_\_\_ 32. Suppose that a 95% confidence interval for  $\mu$  is given by  $\bar{x} \pm 3.25$ . This notation means that, if we repeatedly draw samples of the same size from the same population, 95% of the values of  $\bar{x}$  will be such that  $\mu$  would lie somewhere between  $\bar{x} - 3.25$  and  $\bar{x} + 3.25$ .
- \_\_\_\_\_ 33. The sample size needed to estimate a population mean to within 1 unit with 90% confidence given that the population standard deviation is 10 is 17.
- \_\_\_\_\_ 34. A Type II error is represented by  $\alpha$ ; it is the probability of rejecting a true null hypothesis.
- \_\_\_\_\_ 35. The  $p$ -value of a test is the probability of observing a test statistic at least as extreme as the one computed given that the null hypothesis is true.
- \_\_\_\_\_ 36. A one-tail test for the population mean  $\mu$  produces a test-statistic  $z = -0.75$ . The  $p$ -value associated with the test is 0.7734.

- \_\_\_\_\_ 37. The sampling distribution of  $\hat{p}$  is approximately normal if the sample size is more than 30.

**Short Answer**

38. A sample of 49 measurements of tensile strength for roof hangers are calculated to have a mean of 2.45 and a standard deviation of 0.25. (Units are Newton's per square meter.)
- Determine the 95% confidence interval for mean tensile strength for all hangers.
  - Interpret this confidence interval.
39. An economist is interested in studying the incomes of consumers in a particular region. The population standard deviation is known to be \$1,000. A random sample of 50 individuals resulted in an average income of \$15,000. What is the upper end point in a 99% confidence interval for the average income?

**Statistics Professor**

A statistics professor would like to estimate a population mean to within 40 units with 99% confidence given that the population standard deviation is 200.

40. {Statistics Professor Narrative} What sample size should be used if the standard deviation is changed to 50?
41. Think about a situation where you have a test for a serious disease. First, you are tested positive or negative. Second, you either really do have the disease or you don't.
- If you actually have the disease but the test did not catch it, which error has been made and what is the impact of that error?
  - If you actually don't have the disease but the test says you did, which error is being made and what is the impact of this error?
  - Which error is the worst one to commit in this situation and why?
42. Suppose that 10 observations are drawn from a normal population whose variance is 64. The observations are: 58, 62, 45, 50, 59, 65, 39, 40, 41, and 52. Test at the 10% level of significance to determine if there is enough evidence to conclude that the population mean is greater than 45.

**Hourly Wages**

A random sample of 15 hourly wages for restaurant servers (including tips) was drawn from a normal population. The sample mean and sample standard deviation were  $\bar{x} = \$14.9$  and  $s = \$6.75$ .

43. {Hourly Wages Narrative} Can we infer at the 5% significance level that the mean wage for restaurant servers (including tips) is greater than 12?

**Domino's Pizza**

Domino's Pizza in Big Rapids, Michigan, advertises that they deliver your pizza within 15 minutes of placing an order or it is free. A sample of 25 customers is selected at random. The average delivery time in the sample was 13 minutes with a sample standard deviation of 4 minutes.

44. {Domino's Pizza Narrative} What is the required condition of the technique used in the previous question?
45. Employees in a large company are entitled to 15-minute coffee breaks. A random sample of the duration of coffee breaks for 10 employees was taken with the times shown as: 12, 16, 14, 18, 21, 17, 19, 15, 18, and 16. Assuming that the times are normally distributed, is there enough evidence at the 5% significance level to indicate that on average employees are taking longer coffee breaks than they are entitled to?

**Attorneys**

A random sample of 200 attorneys shows that there are 36 of them who make at least \$400,000 a year.

46. {Attorneys Narrative} Construct a 99% confidence interval estimate of the population proportion of attorneys who make at least \$400,000 a year, and explain how to use it to test the hypotheses.

**Union Contract**

A union composed of several thousand employees is preparing to vote on a new contract. A random sample of 500 employees yielded 320 who planned to vote yes. It is believed that the new contract will receive more than 60% yes votes.

47. {Union Contract Narrative} Compute the  $p$ -value for the test.

**Allergy Drug**

A company claims that 10% of the users of a certain allergy drug experience drowsiness. In clinical studies of this allergy drug, 81 of the 900 subjects experienced drowsiness

48. {Allergy Drug Narrative} Compute the  $p$ -value of the test.

**Car Dealership**

An accountant was performing an audit for a car dealership. An auditor wants to examine the monetary error made by the purchasing order department in the month of July. He decided to randomly sample 100 of the 925 purchase orders for the month of July, and found the amount of error in each one. The statistics for this sample were:  $\bar{x} = \$6.0$  and  $s = \$17.012$ .

49. {Car Dealership Narrative} Estimate with 95% confidence the average amount of error per purchase order for the entire month of July.

## Confidence Intervals and Hypothesis Testing Answer Section

### MULTIPLE CHOICE

- |            |        |                   |
|------------|--------|-------------------|
| 1. ANS: B  | PTS: 1 | REF: SECTION 10.1 |
| 2. ANS: B  | PTS: 1 | REF: SECTION 10.2 |
| 3. ANS: D  | PTS: 1 | REF: SECTION 10.2 |
| 4. ANS: B  | PTS: 1 | REF: SECTION 10.3 |
| 5. ANS: D  | PTS: 1 | REF: SECTION 11.1 |
| 6. ANS: A  | PTS: 1 | REF: SECTION 11.1 |
| 7. ANS: D  | PTS: 1 | REF: SECTION 11.1 |
| 8. ANS: B  | PTS: 1 | REF: SECTION 11.1 |
| 9. ANS: B  | PTS: 1 | REF: SECTION 11.1 |
| 10. ANS: C | PTS: 1 | REF: SECTION 11.2 |
| 11. ANS: C | PTS: 1 | REF: SECTION 11.2 |
| 12. ANS: D | PTS: 1 | REF: SECTION 11.2 |
| 13. ANS: C | PTS: 1 | REF: SECTION 11.2 |
| 14. ANS: D | PTS: 1 | REF: SECTION 11.2 |
| 15. ANS: D | PTS: 1 | REF: SECTION 11.2 |
| 16. ANS: A | PTS: 1 | REF: SECTION 11.2 |
| 17. ANS: C | PTS: 1 | REF: SECTION 11.2 |
| 18. ANS: B | PTS: 1 | REF: SECTION 11.2 |
| 19. ANS: B | PTS: 1 | REF: SECTION 11.2 |
| 20. ANS: B | PTS: 1 | REF: SECTION 11.2 |
| 21. ANS: C | PTS: 1 | REF: SECTION 11.2 |
| 22. ANS: B | PTS: 1 | REF: SECTION 11.3 |
| 23. ANS: D | PTS: 1 | REF: SECTION 11.3 |
| 24. ANS: B | PTS: 1 | REF: SECTION 11.3 |
| 25. ANS: B | PTS: 1 | REF: SECTION 12.1 |
| 26. ANS: C | PTS: 1 | REF: SECTION 12.1 |
| 27. ANS: C | PTS: 1 | REF: SECTION 12.3 |
| 28. ANS: C | PTS: 1 | REF: SECTION 12.3 |
| 29. ANS: B | PTS: 1 | REF: SECTION 12.3 |

### TRUE/FALSE

- |            |        |                   |
|------------|--------|-------------------|
| 30. ANS: T | PTS: 1 | REF: SECTION 10.1 |
| 31. ANS: F | PTS: 1 | REF: SECTION 10.2 |
| 32. ANS: T | PTS: 1 | REF: SECTION 10.2 |
| 33. ANS: F | PTS: 1 | REF: SECTION 10.3 |
| 34. ANS: F | PTS: 1 | REF: SECTION 11.1 |
| 35. ANS: T | PTS: 1 | REF: SECTION 11.2 |
| 36. ANS: F | PTS: 1 | REF: SECTION 11.2 |



37. ANS: F                      PTS: 1                      REF: SECTION 12.3

### SHORT ANSWER

38. ANS:

- a. LCL = 2.38 and UCL = 2.52 (Newton's per square meter).
- b. I estimate the mean tensile strength for all roof hangars to be between 2.38 and 2.52 Newton's per square meter, based on my sample. We know that 95% of the time this method will contain the true population mean.

PTS: 1                      REF: SECTION 10.2

39. ANS:

UCL = \$15,364.87

PTS: 1                      REF: SECTION 10.2

40. ANS:

$n = 11$  (using  $z = 2.58$ )

PTS: 1                      REF: SECTION 10.3

41. ANS:

- a. A Type II error has been committed, which is a very costly error. You are being told you are OK when you really have the disease, and you are going untreated.
- b. This is a false alarm, and a Type I error. A Type I error causes undue worry on behalf of the person taking the test, and could cause some treatments to occur that shouldn't.
- c. With a Type II error you are letting people with the disease go unnoticed, and hence untreated. With a Type I error you falsely tell them they have the disease. This can cause undue worry but it is certainly not as bad of a problem as letting someone go on not knowing they have the disease. A Type II error is the worst in this situation.

PTS: 1                      REF: SECTION 11.1

42. ANS:

$H_0: \mu = 45, H_1: \mu > 45$

Test statistic:  $z = 2.41$

$p$ -value = 0.008

Reject  $H_0$ . Yes, there is enough statistical evidence at the 10% significance level to conclude that the population mean is greater than 45.

PTS: 1                      REF: SECTION 11.2

43. ANS:

$H_0: \mu = 12, H_1: \mu > 12$

Rejection region:  $t > t_{0.05,14} = 1.761$

Test statistic:  $t = 1.664$

Conclusion: Don't reject  $H_0$ . We can't infer at the 5% significance level that the mean wage for restaurant servers (including tips) is greater than 12.

PTS: 1                      REF: SECTION 12.1

44. ANS:  
Delivery times are normally distributed.

PTS: 1 REF: SECTION 12.1

45. ANS:  
 $H_0: \mu = 15, H_1: \mu > 15$   
 Rejection region:  $t > t_{0.05,9} = 1.833$   
 Test statistics:  $t = 1.953$   
 Conclusion: Reject  $H_0$ . Yes, there is enough evidence at the 5% significance level to indicate that on average employees are taking longer coffee breaks than they are entitled to.

PTS: 1 REF: SECTION 12.1

46. ANS:  
 $\hat{p} \pm z_{0.005} \sqrt{\hat{p}(1-\hat{p})/n} = 0.18 \pm 0.07$ . Thus, LCL = 0.11, and UCL = 0.25.  
 Since the hypothesized value  $p_0 = 0.24$  is included in the 99% confidence interval, we fail to reject  $H_0$  at  $\alpha = 0.01$ .

PTS: 1 REF: SECTION 12.3

47. ANS:  
 $p$ -value = 0.0336

PTS: 1 REF: SECTION 12.3

48. ANS:  
 $p$ -value = 0.3174

PTS: 1 REF: SECTION 12.3

49. ANS:  
 $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = 6.0 \pm 1.984 \frac{17.012}{\sqrt{100}} \sqrt{\frac{825}{924}} = 6.0 \pm 3.189$   
 LCL = \$2.81, and UCL = \$9.19 error per purchase order.

PTS: 1 REF: SECTION 12.4-12.5