

MIDTERM 2

Version A

Name: _____

Write a short answer and use graphs to support your argument. Use notations used in class and be sure to label everything if using graphs.

1 Question (20 Points)

Consider an endogenous growth model and suppose that there are three possible uses of time. Let u denote the fraction of time spent working, s the fraction of time being unemployed and doing nothing, and $1 - u - s$ the fraction of time spent accumulating human capital. Assume that the production function is

$$Y = z \times u \times H,$$

where z is the total factor productivity. Human evolves according to

$$H' = b(1 - u - s)H - d \times H,$$

where $d = 10\%$ is the depreciation rate of human capital in every period and b is a production parameter.

- a) Draw a graph of the law of motion for human capital! **(5 points)**
- b) Write down the parameter condition under which the economy can grow. **(5 points)**
- c) Assume $b = 6.0$ and $u = 0.7$. What is the threshold level of unemployment s above which growth will not occur? **(5 points)**
- d) Assume that $z = 2$, $b = 6.0$, $u = 0.7$, and $s = 0.15$. Also assume that the economy begins in period one with $H = 10$ units of human capital. Calculate human capital, consumption, and output for period 4. Be careful with the timing! **(2.5 points)**
- e) Write down the firm problem. Then solve for the firm profit maximizing wage rate. **(2.5 points)**

2 Question (30 Points)

Consider a Solow growth model where the population growth rate is 1%, the constant savings rate is 20%, and the capital depreciation rate is 5%. In addition you can assume a Cobb-Douglas production function in capital and labor with a capital share of production of $\alpha = 0.3$. Suppose that total factor productivity is normalized to 1.

- a) Write down the law of motion of per capita capital holdings and draw a graph of the law of motion. (5 points)
- b) Solve for the long-run steady state level of per capita capital k^* . (5 points)
- c) What is the per capita consumption level in the steady state c^* ? (5 points)
- d) What is the maximum steady state per capita consumption level possible in this economy c_{\max} (Golden Rule)? (10 points)
- e) What is the savings rate s that would result in this maximum per capita consumption level? (5 points)

3 Question (25 Points)

Consumers are endowed with one unit of time in each period and live for two periods. They have the following preferences:

$$u(c, c') = \beta_1 \ln(c) + \beta_2 \ln(c'),$$

where the β coefficients are simply constants and c, c' are current and future consumption. Consumers earn a wage w and w' in current and future periods and pay a proportional payroll tax τ and τ' on labor income in the current and future periods. In addition, consumers can save from the current to the future period and earn interest r on their savings. There are no other sources of income available. The savings of consumers are used as capital on the firm side, so that in equilibrium $S = I = K$ since we assume that the depreciation rate is 100 percent.

- a) Write down the current and future period budget constraints for the representative consumer. **(5 points)**
- b) Write down the lifetime budget constraint of the representative consumer. **(5 points)**
- c) Write down the maximization problem of the consumer without solving it. **(2.5 points)**
- e) Calculate the marginal rate of substitution between current and future consumption. **(5 points)**
- f) What is the price ratio between current and future consumption? **(2.5 points)**
- g) What is the optimality conditions that determines present and future consumption? **(2.5 points)**
- h) What if β_2 is really large compared to β_1 ? Will the consumer be a borrower or a lender? Explain why? **(2.5 points)**

4 Question (25 Points)

Consider the following one period economy. There is a representative household who values consumption and leisure according to $u(c, l)$. The household is endowed with 1 unit of time so that leisure is bounded as $0 < l < 1$. The household can spend part of her time endowment working for which she can earn the wage rate w . However, her labor income is taxed at rate τ (i.e., τ is a proportional payroll tax). In addition, the household inherits a lump-sum amount from a deceased family member which remains tax free. We refer to this bequest (or inheritance) as B . The household lives for one period.

In addition, there is a representative firm that uses a production function that only depends on labor to produce a final consumption good: $Y = z \times F(N_d)$, where z is total factor productivity.

- a) Write down the household budget constraint. **(2.5 points)**
- b) Write down the maximization problem of the consumer without solving it. **(2.5 points)**
- c) Write down the optimality condition of the household. **(2.5 points)**
- d) Write down the maximization problem of the firm. **(2.5 points)**
- e) Write down the optimality condition of the firm. **(2.5 points)**
- f) Write down the definition of competitive equilibrium. **(2.5 points)**
- g) Write down the maximization problem of a social planner. **(5 points)**
- h) Write down the optimality condition of the social planner. **(2.5 points)**
- i) State the first welfare theorem. Does it hold? **(2.5 points)**